U-FNO - an enhanced Fourier neural operator based-deep learning model for multiphase flow

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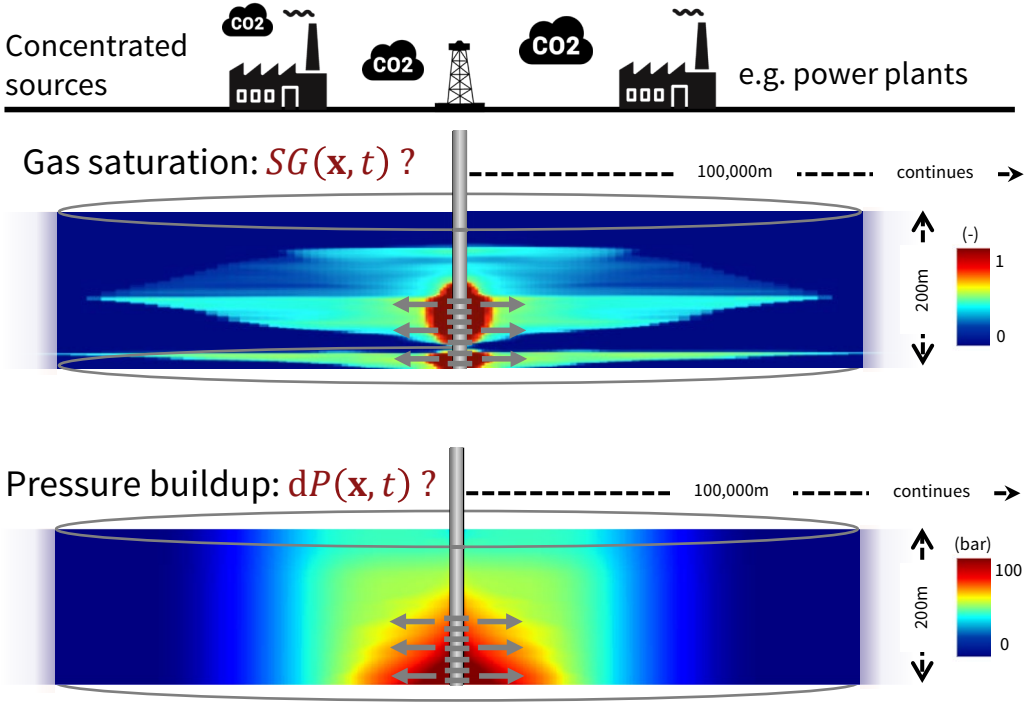
CO_2 storage problem: multiphase flow governed by highly nonlinear PDEs



Mass conservation

$$\frac{\partial M^{\kappa}}{\partial t} = -\nabla \cdot \mathbf{F}^{\kappa} + q^{\kappa} \qquad \kappa = CO_2 \text{ or water}$$

CO_2 component

$$\frac{\partial M^{CO_2}}{\partial t} = -\nabla \cdot \left[\mathbf{F}^{CO_2}\Big|_{adv} + \mathbf{F}^{CO_2}\Big|_{dif} \right] + q^{CO_2}$$

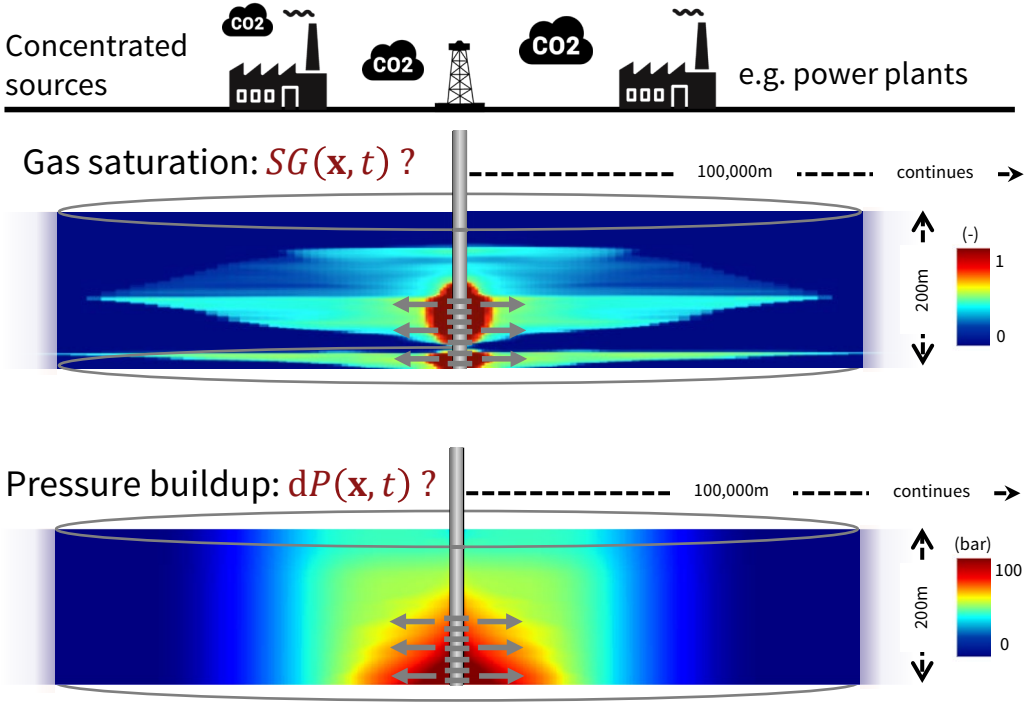
Each phase governed by Darcy's law

$$\frac{\partial \left(\phi \sum_p S_p \rho_p X_p{}^{CO_2}\right)}{\partial t} =$$

$$-\nabla \cdot \left[\sum_p X_p{}^{CO_2} \left(-k \frac{k_{r,p}\rho_p}{\mu_p} \left(\nabla (P + P_c) - \rho_p \mathbf{g} \right) \right) + \mathbf{F}^{CO_2}\Big|_{dif} \right] + q^{CO_2}$$

$$p = liq \text{ or } gas$$

Numerical simulation is the essential tool for solving these PDE, however, very **time consuming**



Key challenges

- Highly nonlinear
- Multi-physics in the problems
- Multiscale heterogeneity
- Need for high grid resolution
- Inherent uncertainty in geology

$$S(\mathbf{x}, t), P(\mathbf{x}, t) = f(\mathbf{x}, t, k(\mathbf{x}), q(\mathbf{x}), Q, T, P_{init}, k_r, P_c, \dots)$$

$\mathbf{x}$	: grid discretization
t	: time
Q	: injection rate
$q(\mathbf{x})$	: injection location
$k(\mathbf{x})$	: permeability map
P_{init}	: initial reservoir pressure
	…

Summary to existing approaches for solving the CO_2 storage subsurface flow problems

Approach	Example	Method	Advantage	Problem
Numerical solver	ECLIPSE	Solve the equation by discretizing the space	PDE-based	Expensive

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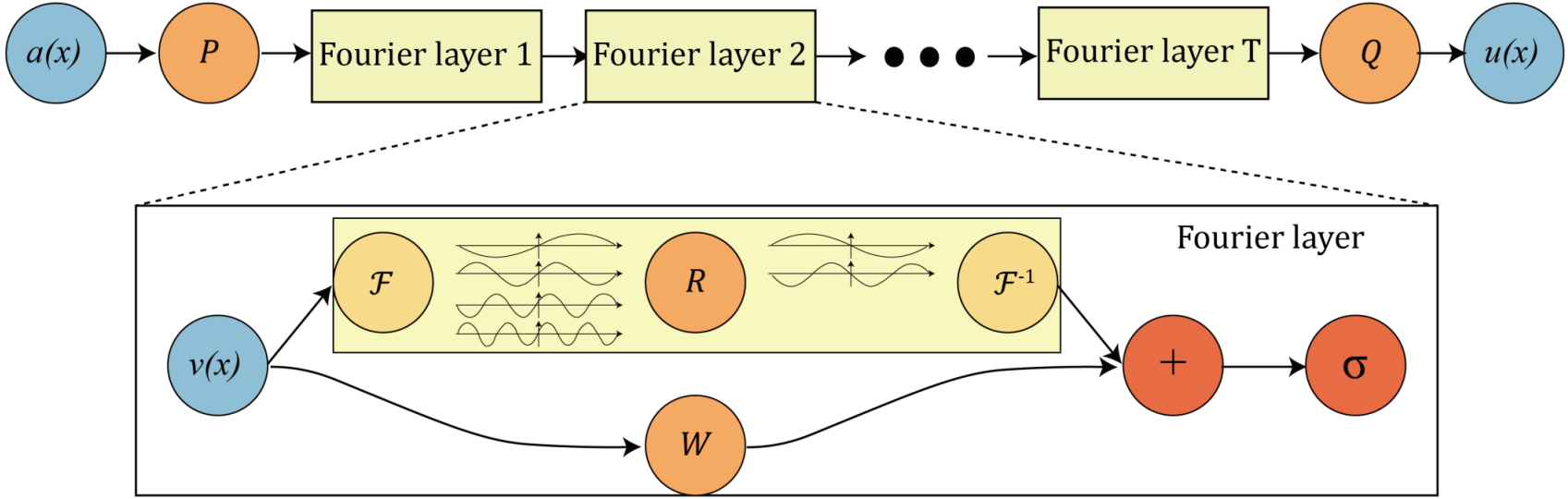
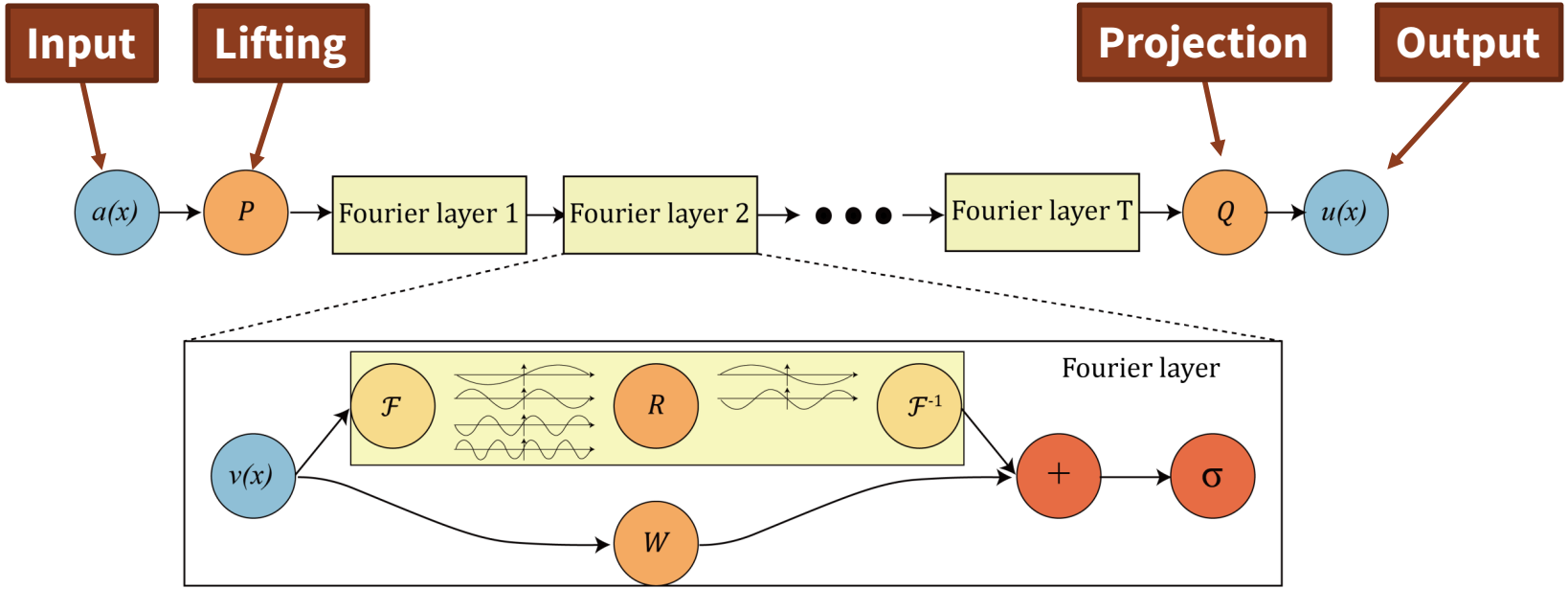
This work: U-FNO

Let's take a closer look at the model architecture of *original* Fourier Neural Operator (Li et al, 2021)



Closer look at the model architecture of the *original* Fourier Neural Operator (Li et al, 2021)



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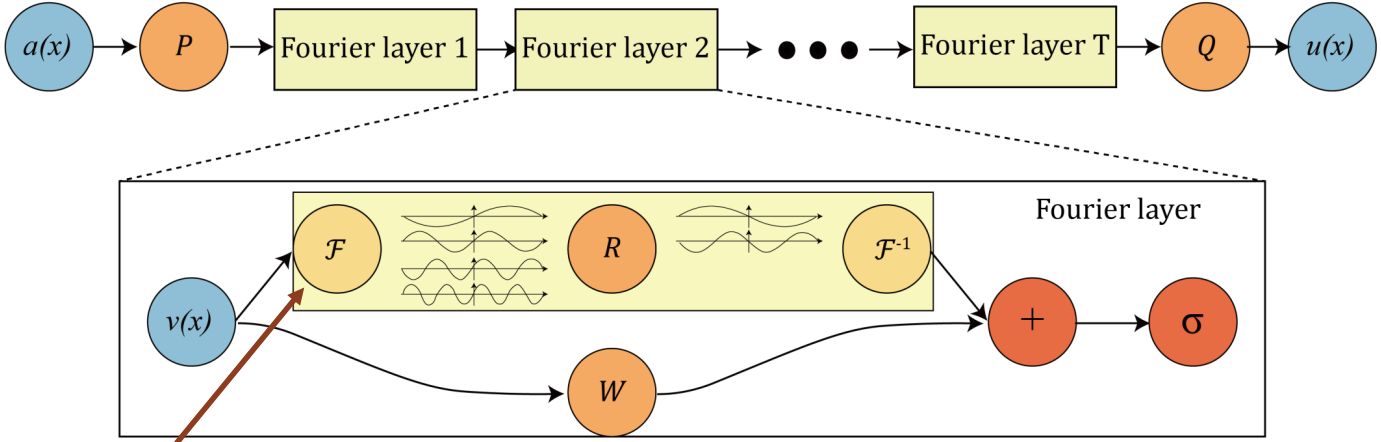

$$(\mathcal{F}f)_j(k) = \int_D f_j(x) e^{-2i\pi\langle x,k\rangle} \mathrm{d}x$$

The transform is conducted utilizing Fast Fourier Transform (FFT).

After the transform, the discrete pixel data becomes **continuous function**.

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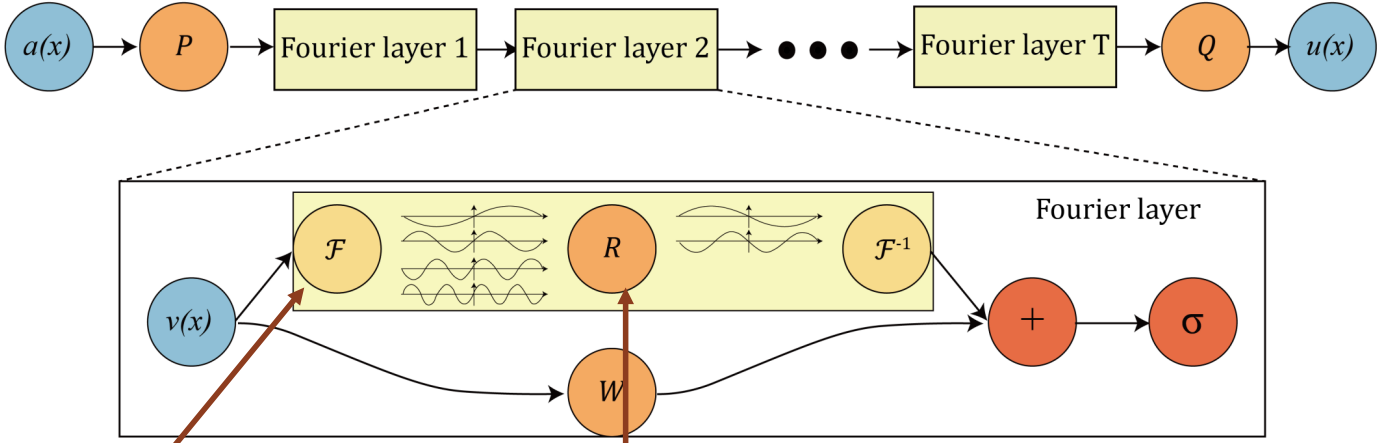

$$\boxed{(\mathcal{K}(\phi)v_t)(x) = \mathcal{F}^{-1}\Big(R_\phi \cdot (\mathcal{F}v_t)\Big)(x)}$$

$$\boxed{(\mathcal{F}f)_j(k) = \int_D f_j(x)e^{-2i\pi\langle x,k\rangle}\mathrm{d}x}$$

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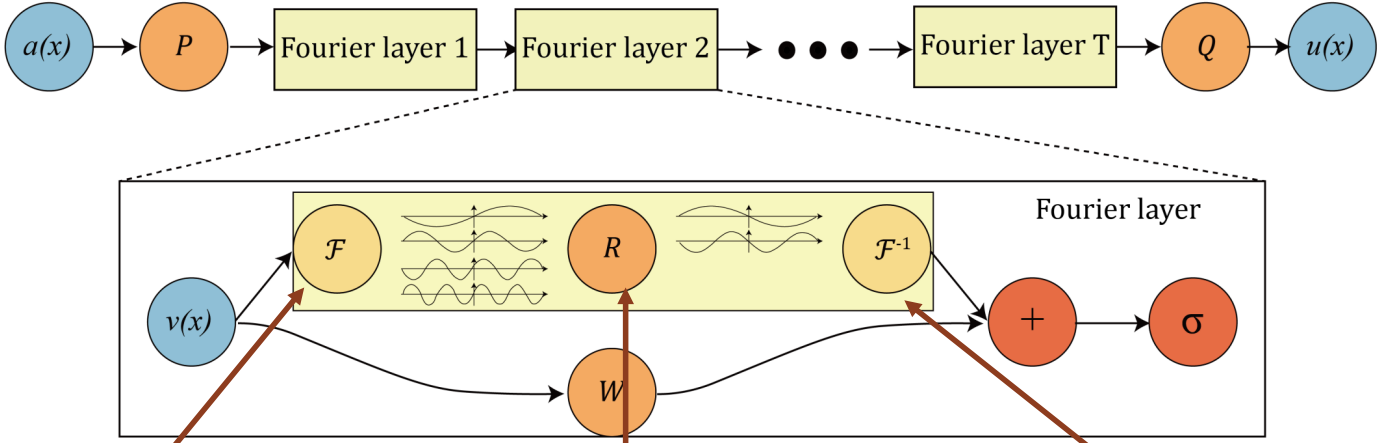
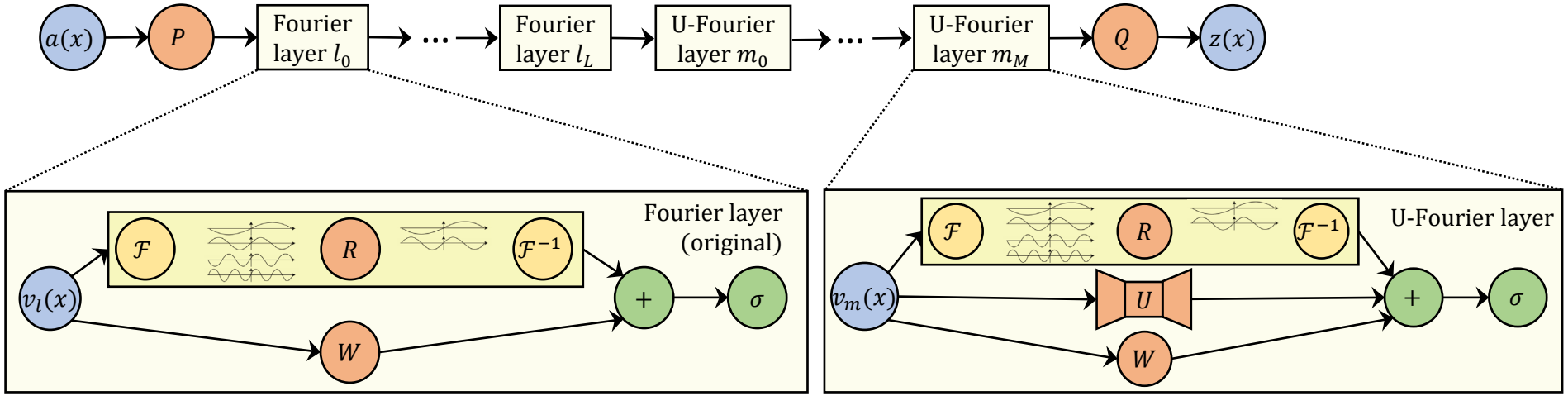


$$(\mathcal{K}(\phi)v_t)(x) = \mathcal{F}^{-1}\Big(R_\phi \cdot (\mathcal{F}v_t)\Big)(x)$$

$$(\mathcal{F}f)_j(k) = \int_D f_j(x)e^{-2i\pi\langle x,k\rangle}\mathrm{d}x$$

$$(\mathcal{F}^{-1}f)_j(x) = \int_D f_j(k)e^{2i\pi\langle x,k\rangle}\mathrm{d}k$$

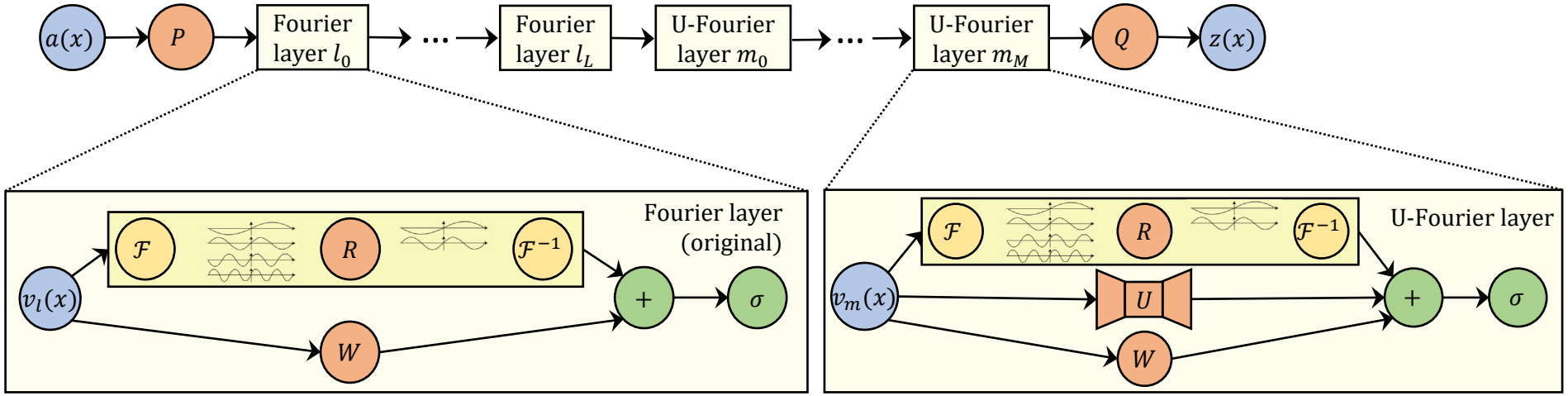
U-FNO Architecture: enhanced Fourier Neural Operator for solving the CO_2-water multiphase flow problem



Remark 1:
2-step mini U-Net to enhance higher frequency information

U-FNO Architecture: enhanced Fourier Neural Operator for solving the CO_2-water multiphase flow problem

Remark 2: L Fourier layers followed by M U-Fourier layers, L and M are hyper-parameters



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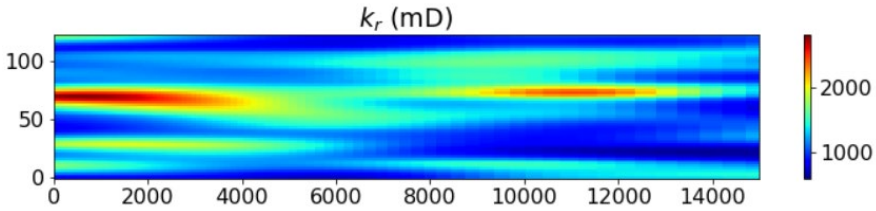
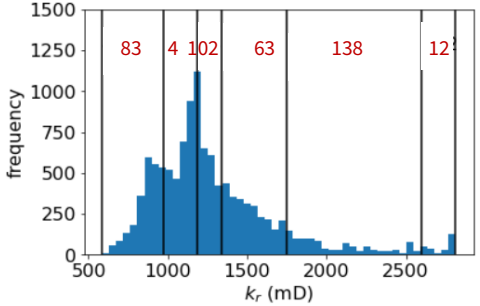
2-step mini U-Net to enhance higher frequency information

Anisotropic permeability and heterogeneous porosity data generation recipe

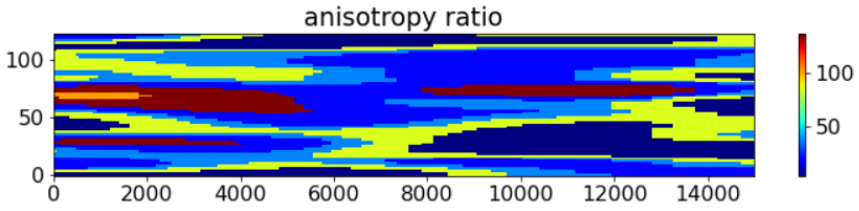
Step 1. Start with radial permeability map



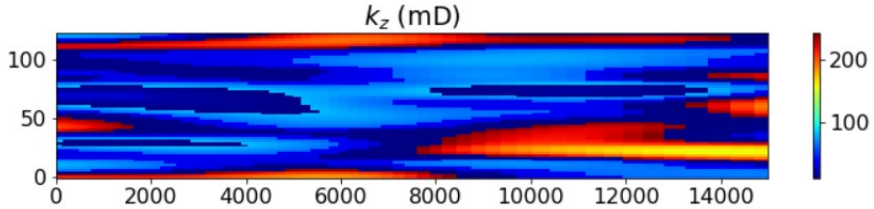
Step 2. sample the random # of anisotropic materials (1-6) and divide the radial perm distribution to # of materials



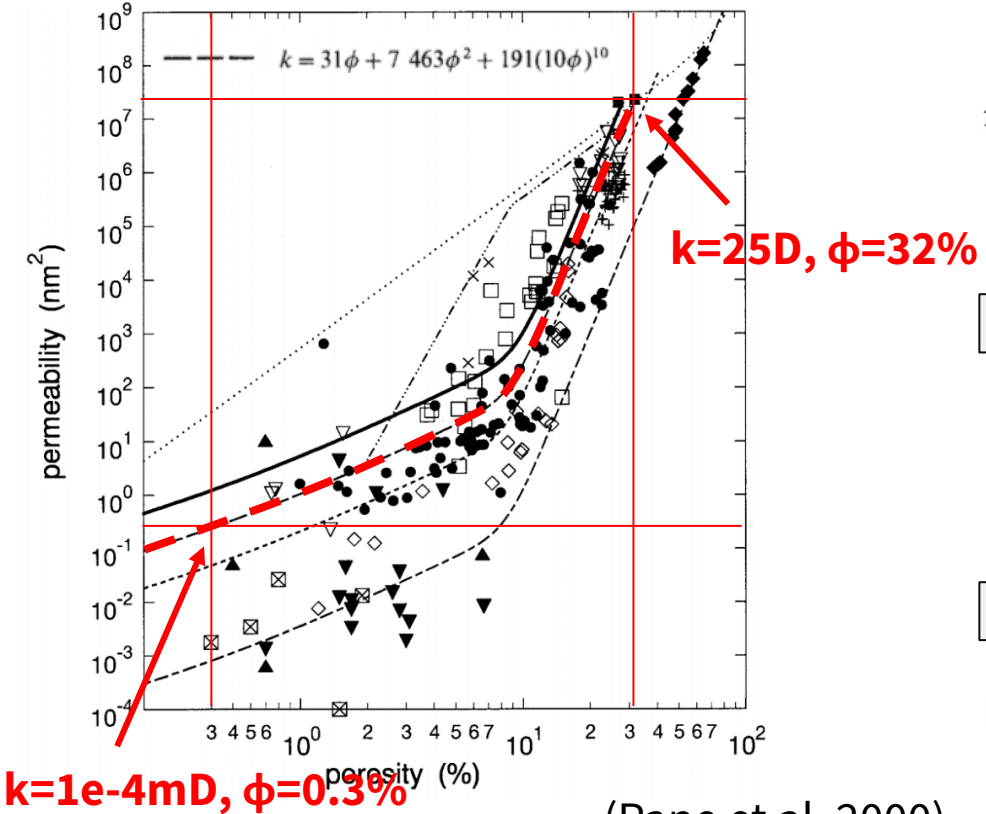
Step 3. assign each material to a random anisotropy ratio (1-150)



Step 4. calculate vertical perm using anisotropy ratio

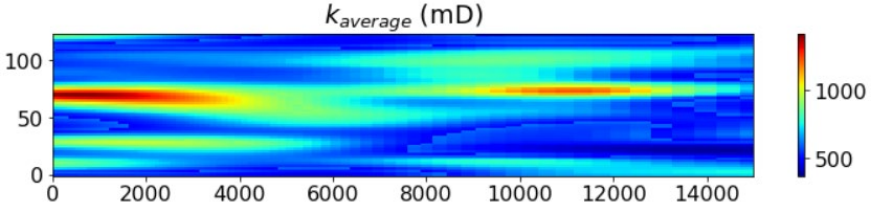


Create porosity field based on experimental relationships

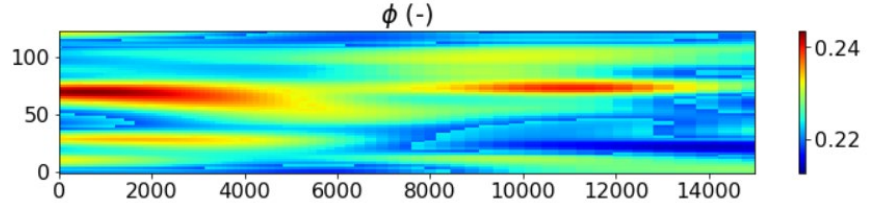


(Pape et al, 2000)

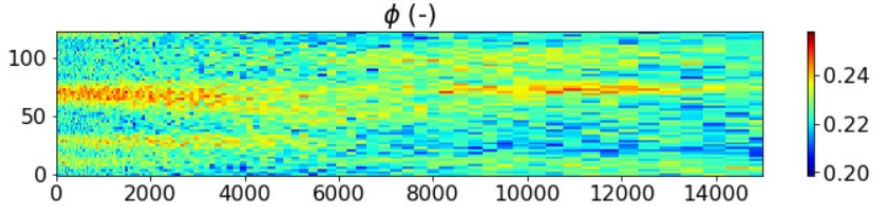
Step 1. Calculate permeability average



Step 2. Solve for porosity using permeability average



Step 3. Add a random noise ε with std=0.005



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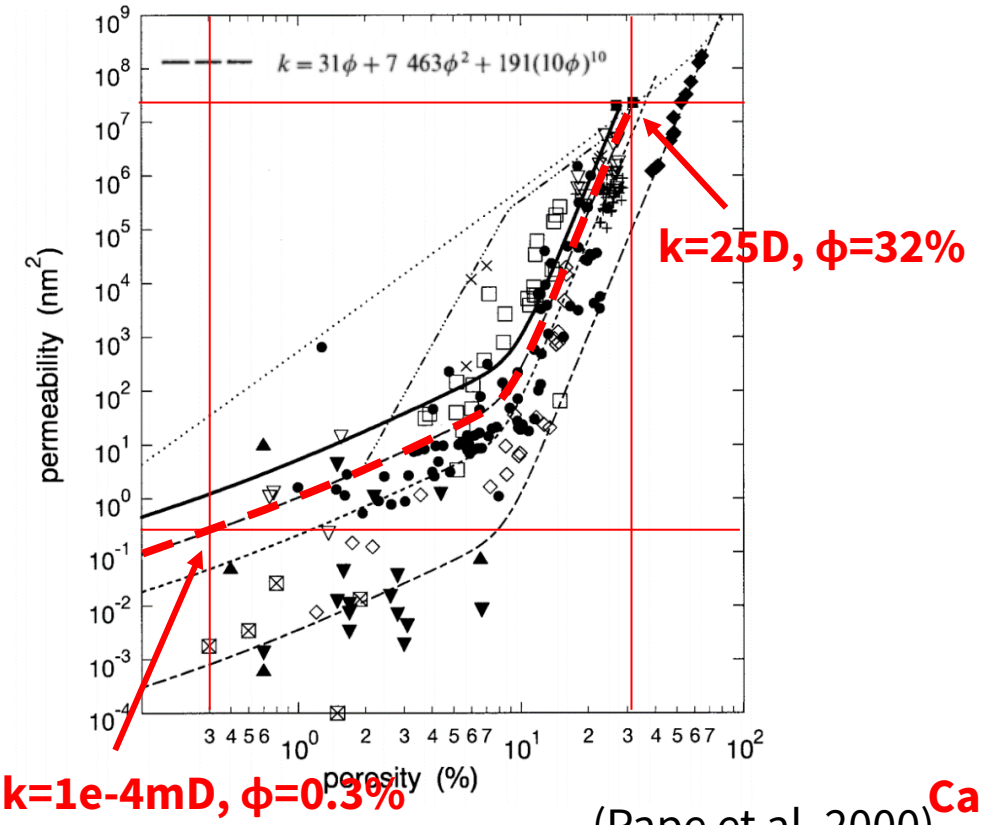

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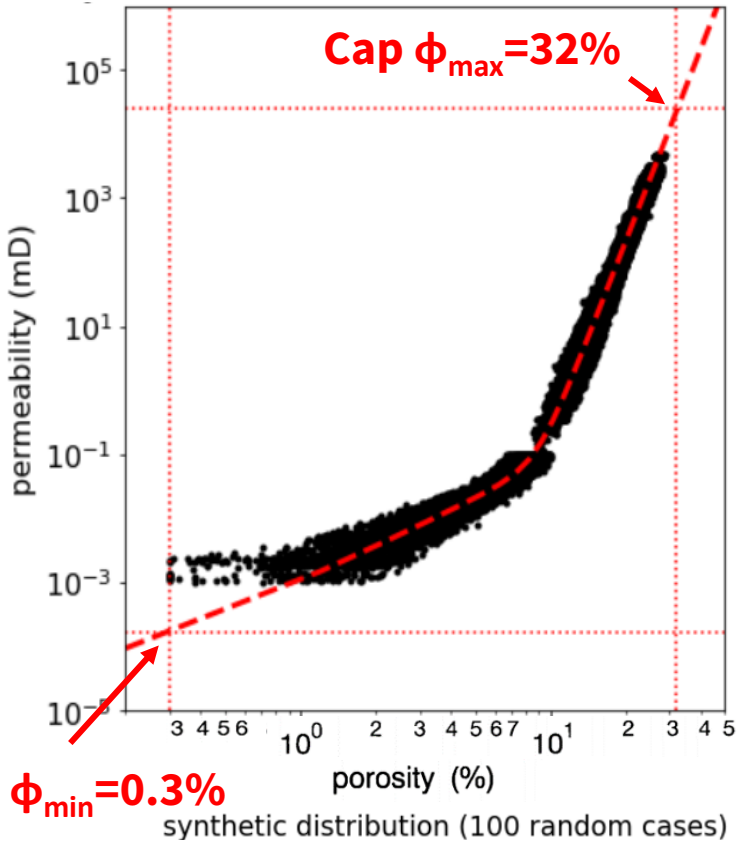
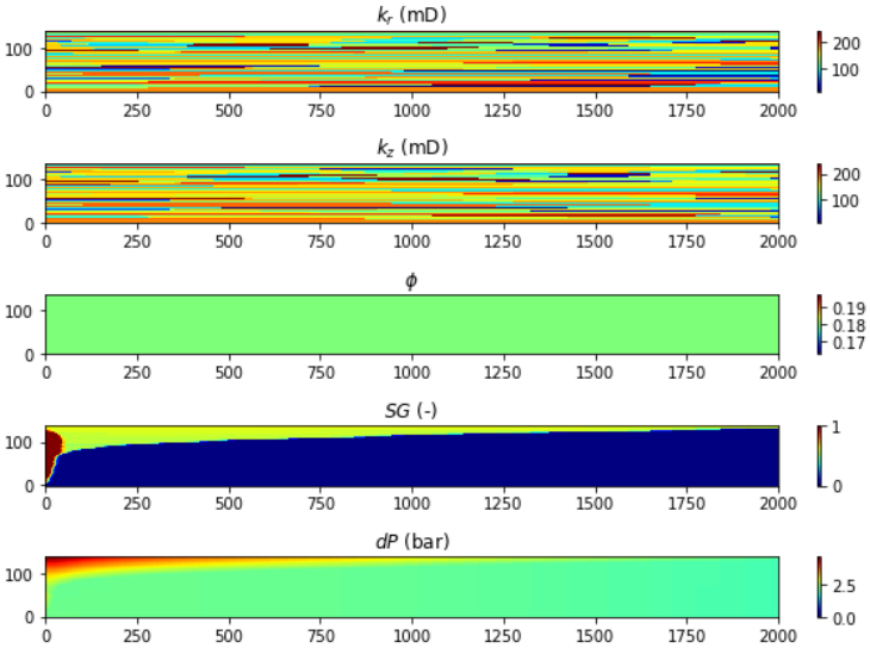


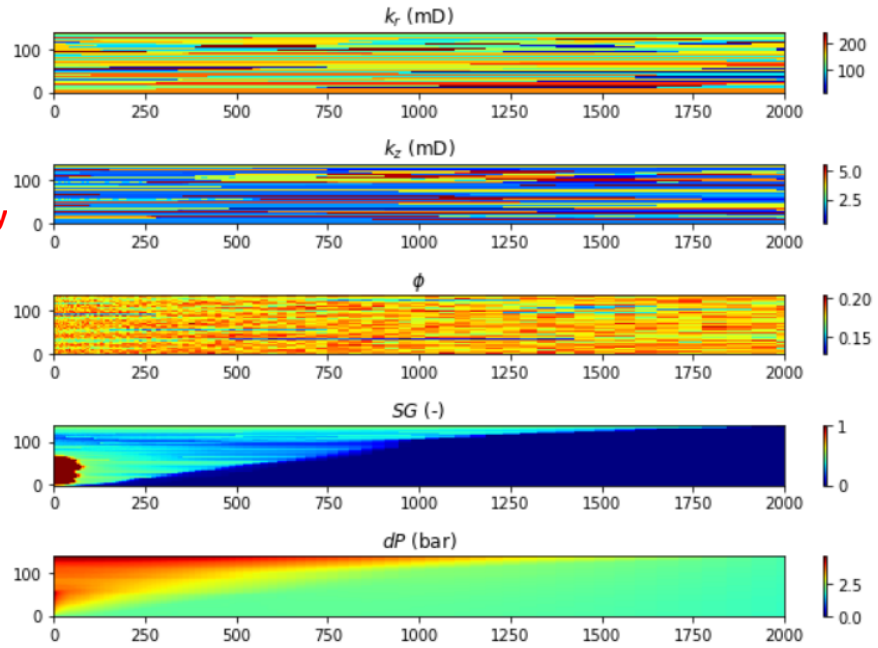
Adding porosity and anisotropy increase the input dimension for 38,000 times



CCSNet dataset

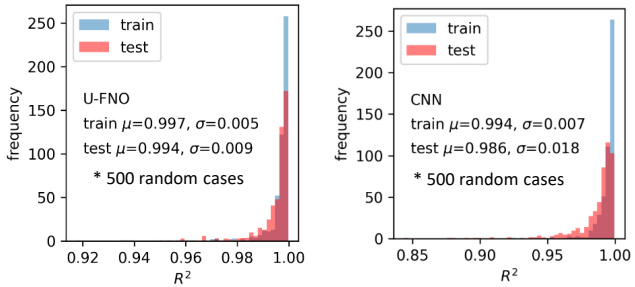
anisotropy

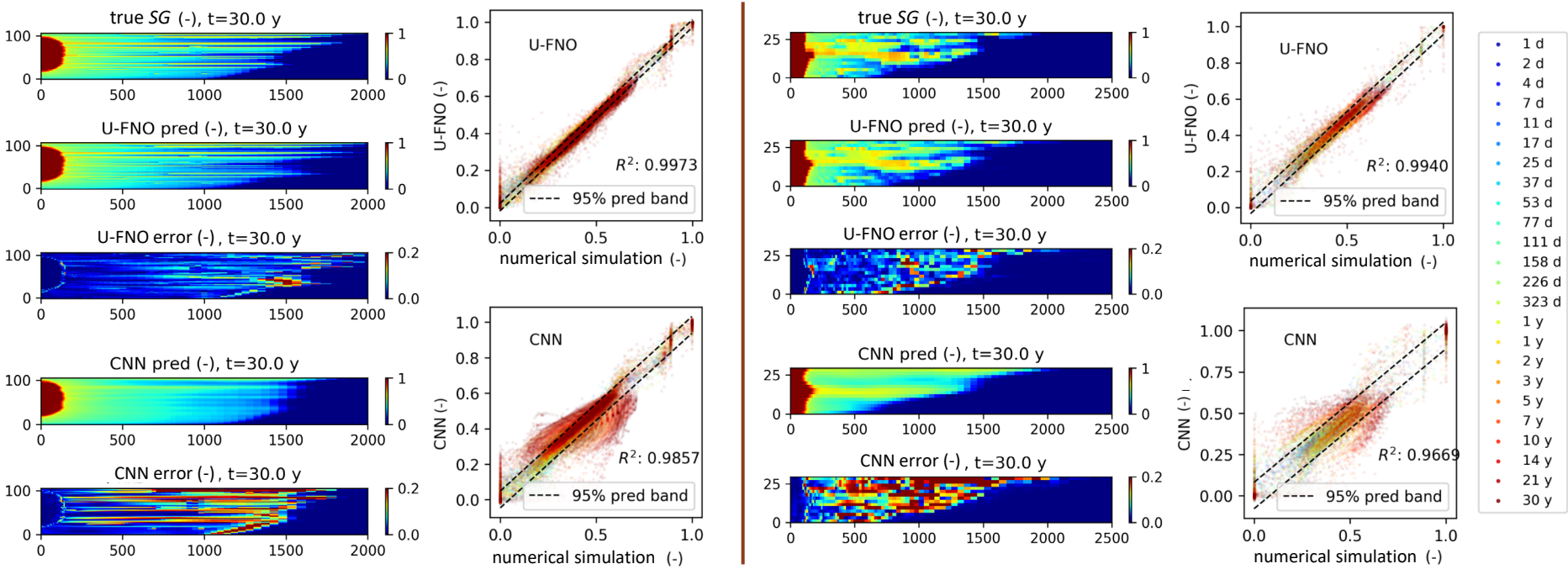
porosity



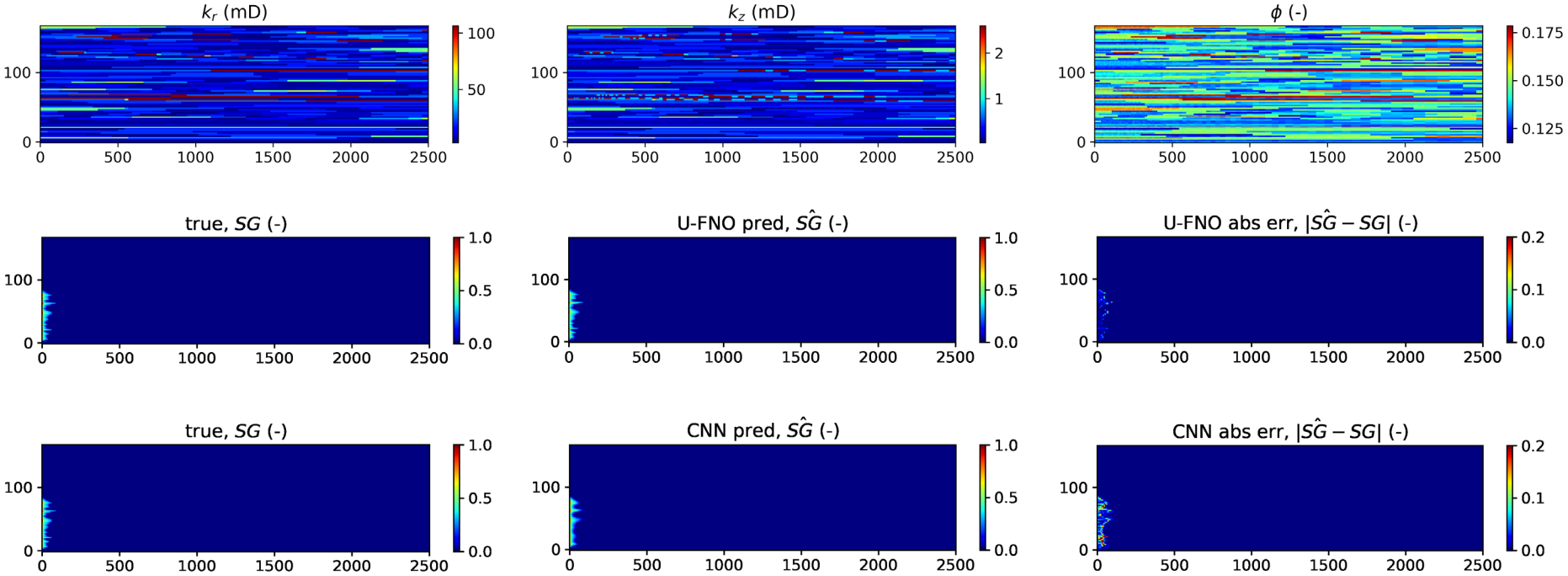
New dataset

Result: CO_2 gas saturation plume prediction greatly improved with U-FNO comparing to CNN

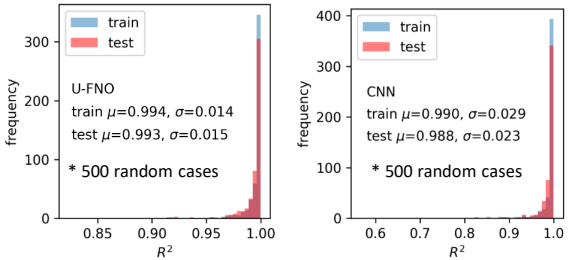


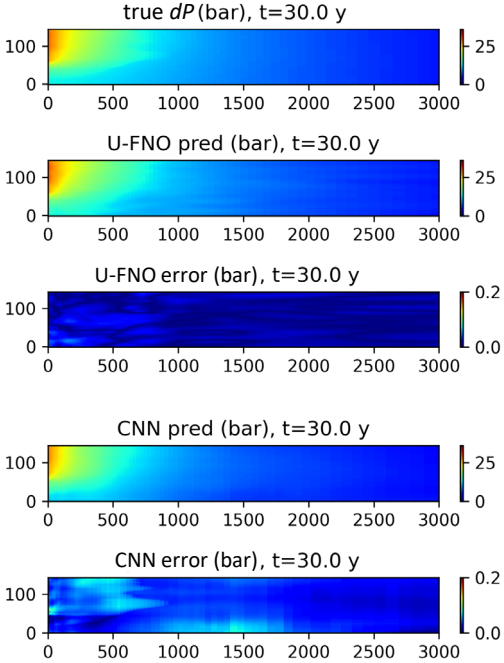


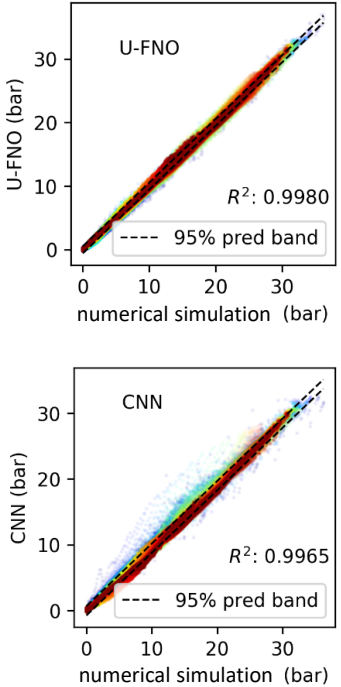
Result: U-FNO produces consistently superior CO_2 gas saturation plume prediction throughout the injection period

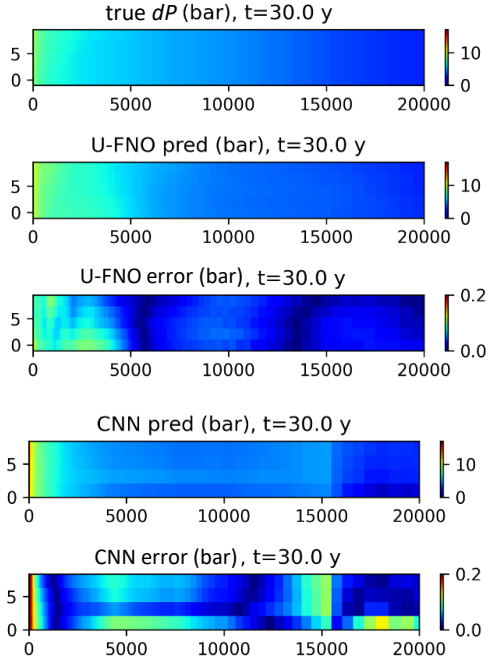


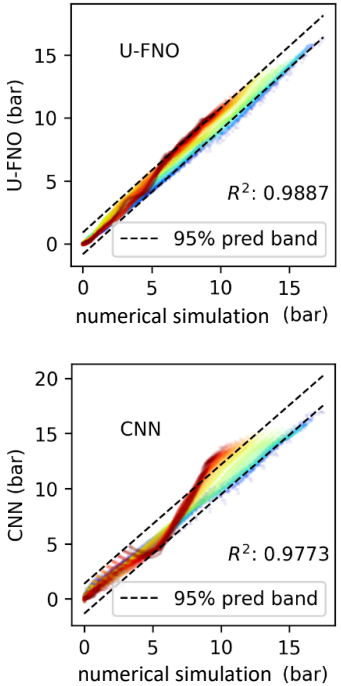
Result: Pressure buildup prediction greatly exceeded CNN; low pressure cases significantly improved

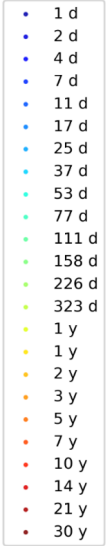




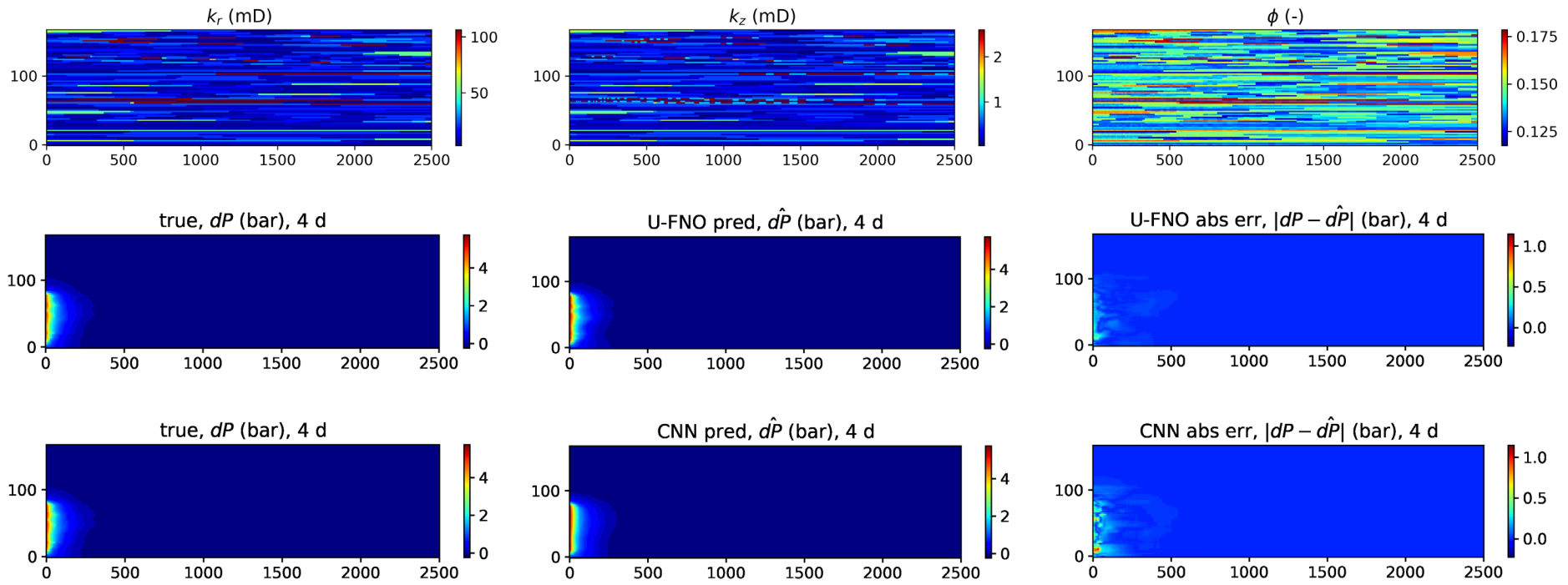






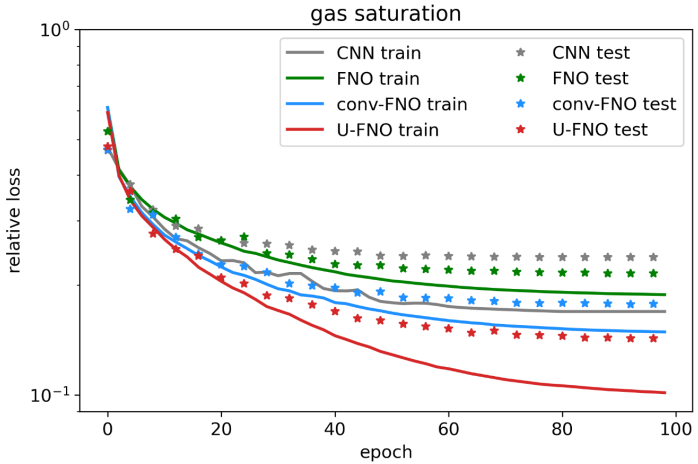


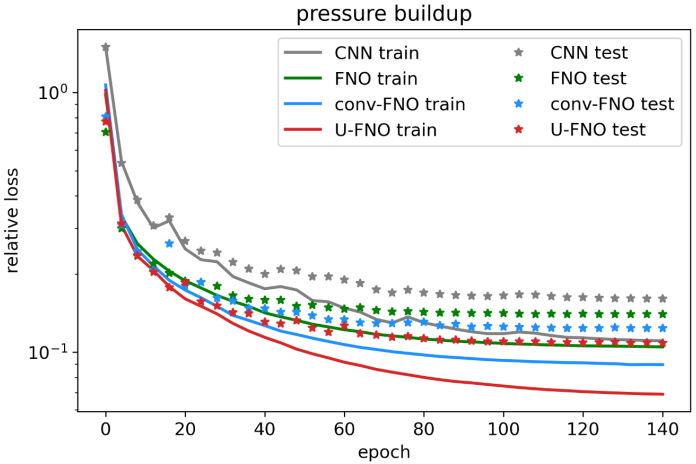
Result: U-FNO produces consistently superior pressure buildup prediction throughout the injection period



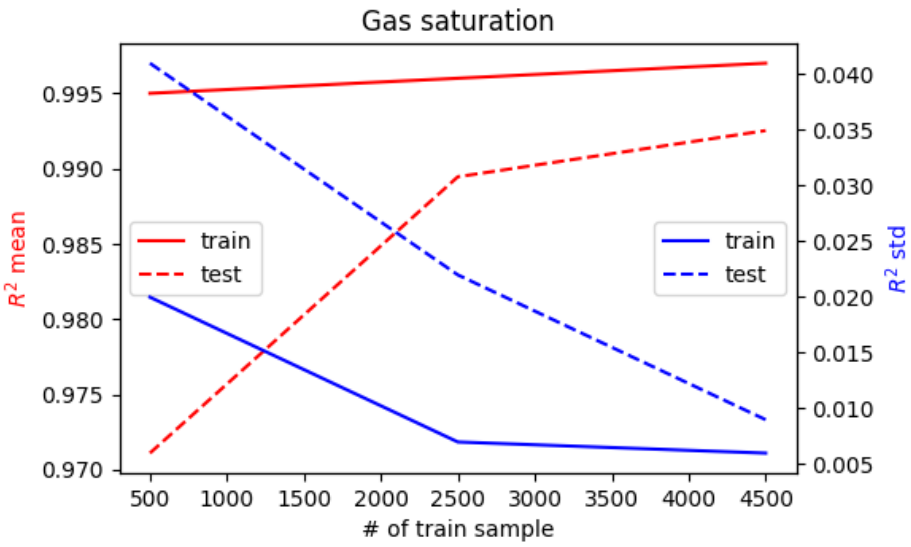
Result: U-FNO has superior performance on both training and testing sets among CNN and original FNO

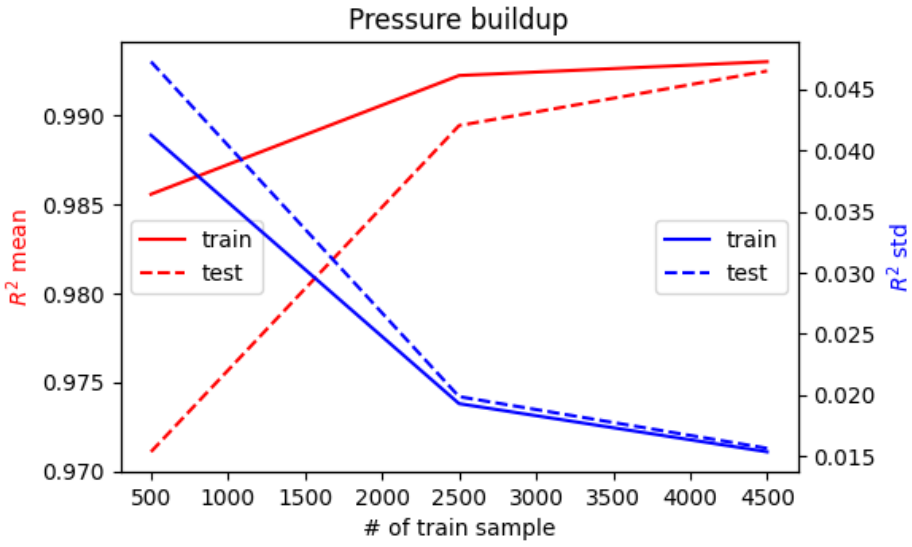
Model	Gas saturation (SG) R2				Pressure buildup (dP) R2			
	Train		Test		Train		Test	
	mean	std	mean	std	mean	std	mean	std
CNN	0.994	0.007	0.986	0.018	0.990	0.029	0.988	0.023
FNO	0.990	0.013	0.986	0.017	0.991	0.014	0.990	0.017
Conv-FNO	0.993	0.011	0.990	0.013	0.992	0.018	0.991	0.018
U-FNO	**0.997**	**0.005**	**0.994**	**0.009**	**0.994**	**0.014**	**0.993**	**0.015**





Remark: great data efficiency – more complex physics, less data required





U-FNO only requires 4500 data samples for training
CCSNet requires 20000 data samples to achieve similar performance

Computational efficiency: prediction speed up is 60000x vs. numerical simulation; even faster than CNN

Table 3: Summary of the number of parameters, training time, and testing times required for all four models. The testing times are calculated by taking the average of 500 random cases. The speed-up is compared with average numerical simulation run time of 10 mins.

	# Parameter (-)	Training (s/epoch)	Testing		
			Gas saturation (s)	Pressure buildup (s)	Speed-up vs. numerical simulation (times)
CNN	33,316,481	562	0.050	0.050	1×10^4
FNO	31,117,541	711	0.005	0.005	1×10^5
Conv-FNO	31,222,625	1,135	0.006	0.006	1×10^5
U-FNO	33,097,829	1,872	0.010	0.010	6×10^4

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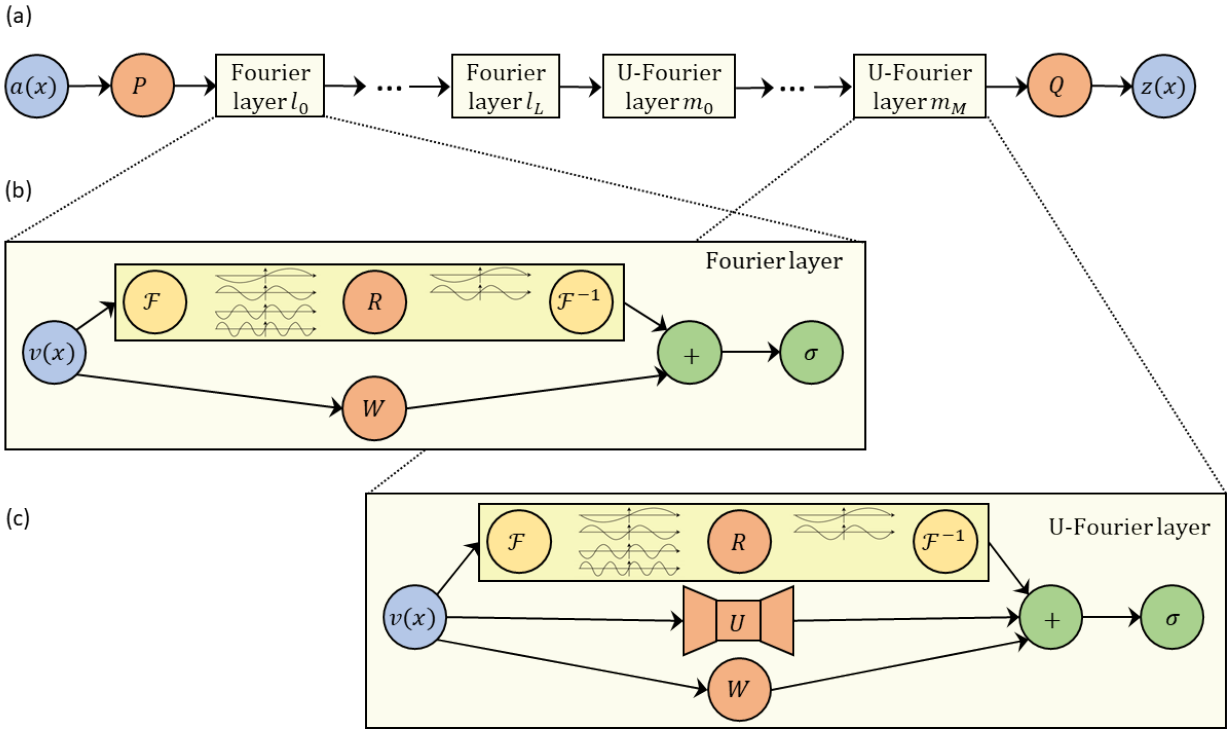

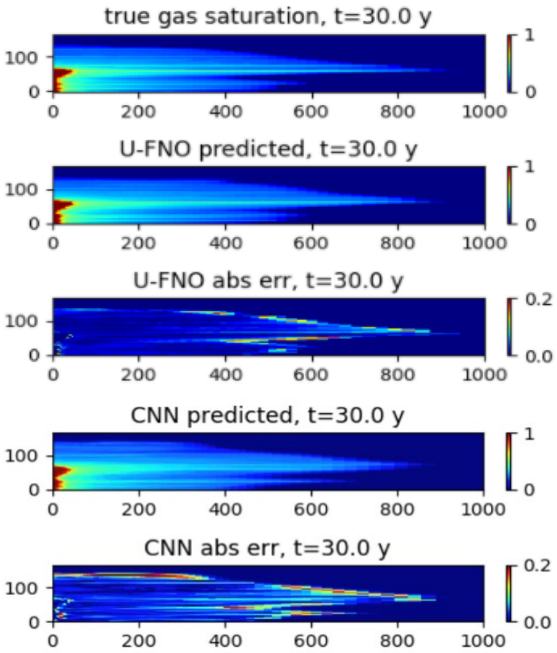

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