Application of Reduced-Order Models for Optimization in CO$_2$ Storage

Zhaoyang Larry Jin & Louis J. Durlofsky

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Motivation: CO$_2$ Storage

- Optimize well controls to reduce risk
- Minimize target layer CO$_2$ mobility/molar fraction

Base Case

Optimized Solution
Outline

- Basic POD-TPWL formulation for CO$_2$ storage
- Use of derivatives from multiple training runs
- Application in well-control optimization
- Summary and future work
Proper Orthogonal Decomposition

- Take high-fidelity snapshots to form $X_z$ and $X_p$.

\[
X_z = \begin{bmatrix} z^1 & z^2 & \cdots & z^L \end{bmatrix}, \quad X_p = \begin{bmatrix} p^1 & p^2 & \cdots & p^L \end{bmatrix}
\]
Proper Orthogonal Decomposition

- SVD: \( X_p = U_p \Sigma_p V_p^T \) and \( X_z = U_z \Sigma_z V_z^T \)
- Leading \( l_p, l_z \) columns of \((U_p, U_z)\) give \((\Phi_p, \Phi_z)\)

\[
\begin{bmatrix}
X_p \\
X_z
\end{bmatrix}
= \begin{bmatrix}
\Phi_p & 0 \\
0 & \Phi_z
\end{bmatrix}
\begin{bmatrix}
\xi_p \\
\xi_z
\end{bmatrix}
\]

write as: \( x = \Phi \xi \)

\[\text{dim}(x) = 2n_b \sim O(10^4 - 10^6)\]

\[\text{dim}(\xi) = l = l_p + l_z \sim O(10^2 - 10^3)\]
Trajectory Piecewise Linearization

2D state space
First order accuracy

Cardoso (2010)
POD-TPWL Formulation (1)

- Linearized representation for new state $x^{n+1}$

$$g^{n+1} = 0 \approx g^{i+1} + \frac{\partial g^{i+1}}{\partial x^{i+1}}(x^{n+1} - x^{i+1}) + \frac{\partial g^{i+1}}{\partial x^i}(x^n - x^i) + \frac{\partial g^{i+1}}{\partial u^{i+1}}(u^{n+1} - u^{i+1})$$

- Introduce POD basis matrix $\Phi$

$$x \approx \Phi \xi \quad \xi: \text{reduced } p \text{ and } z \text{ variables, } l\text{-dimensions}$$

$x$: states $(p, z_w)$  $u$: controls (BHPs, rates)

training run: $(i, i+1)$  test run: $(n, n+1)$

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POD-TPWL Formulation (2)

- Apply Proper Orthogonal Decomposition (POD)

\[
\frac{\partial g^{i+1}}{\partial x^{i+1}} \Phi(\xi^{n+1} - \xi^{i+1}) = - \left[ \frac{\partial g^{i+1}}{\partial x^i} \Phi(\xi^n - \xi^i) + \frac{\partial g^{i+1}}{\partial u^{i+1}} (u^{n+1} - u^{i+1}) \right]
\]

- Constraint reduction: pre-multiply by \((\Psi^{i+1})^T = (J^{i+1} \Phi)^T\)

\[
J_r^{i+1}(\xi^{n+1} - \xi^{i+1}) = - \left[ A_r^{i+1}(\xi^n - \xi^i) + U_r^{i+1}(u^{n+1} - u^{i+1}) \right]
\]

\[
J_r^{i+1} = (\Psi^{i+1})^T \frac{\partial g^{i+1}}{\partial x^{i+1}} \Phi, \quad A_r^{i+1} = (\Psi^{i+1})^T \frac{\partial g^{i+1}}{\partial x^i} \Phi, \quad U_r^{i+1} = (\Psi^{i+1})^T \frac{\partial g^{i+1}}{\partial u^{i+1}}
\]

(after Cardoso 2010; He 2014)
Derivatives from Multiple Trainings (1)

- Additional trainings with derivatives
- More sophisticated point selection criteria

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Derivatives from Multiple Trainings (2)

- Selection of both training run $\tau$ and point $j$

\[
d = d_z + \gamma d_T
\]

\[
d_z = d_z(n, j, \tau) = \frac{|\xi^n_z - \xi^{j,\tau}_z|}{|\xi^n_z| + \epsilon}, \quad \xi : \text{reduced states}
\]

\[
d_T = d_T(n, j, \tau) = \sum_{k=1}^{n_w} \frac{\left| \int_0^{t^n} q^n_k \, dt - \int_0^{t^j} q^{j,\tau}_k \, dt \right|}{\left| \int_0^{t^n} q^n_k \, dt \right| + \epsilon}
\]

\[
d(n, j^*, \tau^*) = \min_{j \in D_{\tau,\tau}=1,...n_t} d(n, j, \tau)
\]
POD-TPWL Flow Chart

Online Process
- Test controls
  - Single Derivative

- POD-TPWL
  - Reduced derivatives: \( \{J^i_r\} \)
  - Reduced states: \( \{\xi^i\}_1, \{\xi^i\}_2, \ldots, \{\xi^i\}_5 \)

Offline Process
- Training controls
  - AD-GPRS (HFS)

Post Processing
- Primary Output
  - Flash
- Secondary Output

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POD-TPWL Flow Chart

Online Process

- Test controls
- POD-TPWL*

Multiple Derivatives

- Reduced derivatives: \( \{J^i_r\}_1, \{J^i_r\}_2, \ldots, \{J^i_r\}_5 \)
- Reduced states: \( \{\xi^i\}_1, \{\xi^i\}_2, \ldots, \{\xi^i\}_5 \)

Offline Process

- Training controls
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Post Processing

- Primary Output
  - Flash
  - Secondary Output

* POD-TPWL implemented as C++ code

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Test Model Setup

- Gaussian log-permeability, low-perm at layer 7
- Full model: 43 km x 43 km x 150 m
- Storage aquifer: 2.8 km x 2.8 km x 150 m
- Storage aquifer model: 35 x 35 x 15 blocks
Training and Test Rates

- Field injection rate: 0.3 million tonnes of CO$_2$ per year
Training and Test Rates

- Field injection rate: 0.3 million tonnes of CO₂ per year

Training Well 3

Test Well 3
Test Case 1: BHP for Well 2

Single derivative

- Single derivative uses snapshots from 5 trainings
- POD-TPWL speed up $\sim 100-150x$

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Test Case 1: BHP for Well 2

Single derivative

- Single derivative uses snapshots from 5 trainings
- POD-TPWL speed up ~100-150x

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Test Case 1: BHP for Well 3

- Single derivative uses snapshots from 5 trainings
- POD-TPWL speed up ~100-150x

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CO$_2$ Molar Fractions at Target Layer

Training (full order)  Test (full order)

T = 7300 days
CO₂ Molar Fractions at Target Layer

Single derivative

|TPWL – HFS| (test)

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Aggregate CO$_2$ at Target Layer

Aggregate CO$_2$: \( J = \sum_{i\in D_{cl}} z_i(T) \)

\( D_{cl} \) : target layer blocks
\( T = 7300 \) days
Optimization Methodology

- Minimize aggregate CO$_2$ at target layer
  - Combine MADS with POD-TPWL
  - Use of multiple derivatives to avoid re-training
Optimization Model Setup (1)

- Channelized aquifer derived from Stanford VI
- Full model: 232 km x 232 km x 100 m
- Storage aquifer: 10.9 km x 10.9 km x 100 m
- Storage aquifer model: 25 x 25 x 10 blocks

log permeability

4 horizontal wells

Full model

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Optimization Model Setup (2)

- Time-varying rate controls
- Fixed field injection rate
- Simulation time 20 years
- Cumulative CO$_2$ injected: 2.8% of total pore volume
- 5 training runs for POD-TPWL

Well controls for initial guess
Optimization Formulation

- **Control variables:** injection rates at 4 wells for each period, total of 16 variables, 12 free variables

- **Optimization constraint:** constant field injection rate

- **Objective function:** aggregate CO$_2$ in the target layer (layer 3)

\[
J(u) = \sum_{i \in D_{cl}} z_i (T = 20 \text{ yrs})
\]

- **MADS algorithm (NOMAD), 200 function evaluations**
Optimization Results (1)

![Graph showing the decrease of Aggregate CO₂ with the number of simulations. The line indicates a decrease and is labeled Full-Order.]
Optimization Results (1)

![Graph showing the comparison between Full-Order and Reduced-Order simulations of Aggregate CO₂ over the number of simulations. The graph indicates a decrease in Aggregate CO₂ as the number of simulations increases, with the Reduced-Order method converging closer to the Full-Order method with a lower number of simulations.](image)
Optimization Results (1)

![Graph showing optimization results](image)

- Full-Order
- Reduced-Order
- Validation Points

Number of simulations

Aggregate CO₂
Optimization Results (2)

- Full-order: 0.98
- Best POD-TPWL: 0.96
CO$_2$ at Layer 3

Initial guess

Optimized solution (HFS)

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Optimized CO$_2$ Field

Initial guess, well 3

Optimized solution, well 3
Optimized CO₂ Field

Initial guess, well 3

Optimized solution, well 3

Initial guess, well 1

Optimized solution, well 1

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Error in Optimization Solution

|TPWL – HFS|, well 3

|TPWL – HFS|, well 1

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Summary

- POD-TPWL appears to be capable of predicting relevant quantities for carbon storage
  - CO₂ plume (molar fraction at target layer)
  - Well BHP response

- Accuracy improved by introducing derivatives from multiple training runs

- Optimization workflow has been developed, reasonable results achieved
Future Work

- Refine optimization procedure (consider use of Stanford Unified Optimization Framework)
- Extend POD-TPWL for use in equilibration stage of carbon storage operations
- Apply POD-TPWL for uncertainty quantification
- Enhance POD-TPWL to use localized bases
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