Nonlinearity Analysis for Two-Phase Transport in Porous Media

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Introduction

- Long-term objective:
  - Build an efficient nonlinear solver for coupled flow and transport in porous media, applicable in modeling CO₂ storage, oil recovery, etc.

- Previous works:
  - Potential ordering strategy
    (Kwok & Tchelepi, JCP 2007)
  - Localization technique
    (Lu and Beckner, SPE 2011)
  - Ordering with localization
    (Shahvali & Tchelepi, SPE 2013)
  - Trust-Region Chopping
    (Jenny et al., JCP 2009; Wang & Tchelepi, JCP 2013)

- Here, we offer a deep theoretical probe into the nonlinearity of the simulation problem
Why Non-convergence Occurs?

Due to the inflection point:

Due to the kink:

Solution

Residual function

Solution

Trust-Region Chopping Method

[1] Jenny et al., 2009
[3] Li & Tchelepi, 2014
Numerical Flux (F)

Single-point upstream weighting is used.
We consider viscous and buoyancy forces here.

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Numerical Flux (F)

Derivative w.r.t. $S_{\text{WL}}$

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Theory of Inflection Lines

- For viscous and buoyancy forces ($Ng = -5$):

![Diagram showing numerical flux and inflection lines](image1)

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Inflection Lines on Numerical Flux

- Buoyancy is stronger ($Ng = -10$)

![Diagram showing numerical flux and inflection lines](image2)

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Inflection Lines on Numerical Flux

- Gravity segregation ($Ng = -\infty$):

![Numerical Flux (F) diagram]

Chopping Strategies

- Locate the kink and inflection lines:

![Chopping Strategies diagram]
Numerical Example

Example: 1D gravity segregation

Initial condition:

\[ g \]

\[ \begin{align*}
\text{Sw} &= 1 \\
\text{Sw} &= 0
\end{align*} \]

80 cells

Convergence Analysis

- An example of convergence failure
Convergence Analysis

Our method:

![Convergence Graph](image)

2D Heterogeneous Model

Initial condition:

- Water (heavier)
- Oil (lighter)

Injection rate: 1.4 CPVI / day

Logarithm of permeability $\log_{10}(k)$ in md

- Injector
- Producer

Initial condition:

- 0 day
- 200 days
- 2000 days
2D Heterogeneous Model

Nonlinearity of Capillarity

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New Scheme

For viscous, buoyancy, and heterogeneous capillary forces:

Numerical Example

- Panel (a): Graph showing the distribution of a parameter across a dimensionless coordinate.
- Panel (b): Graph depicting the relationship between another parameter and a dimensionless variable.
- Panel (c): Graph illustrating the evolution of a parameter with respect to a dimensionless coordinate over time.
Numerical Example

Conclusions

- The nonlinearity of the simulation problem can be understood by studying the nonlinear space of each cell interface
- Kinks and inflection lines cause convergence failure. They can be accurately located
- Our nonlinear solver guides Newton iterations to progress through kinks and inflection lines without causing oscillation and divergence
- Convergence is achieved for timesteps spanning several orders of magnitude
- A clear understanding of the nonlinearity sparks innovations of nonlinear solvers and numerical schemes
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