Nonlinearity Analysis for Two-Phase Transport in Porous Media

Boxiao Li
Sally M. Benson
Hamdi A. Tchelepi

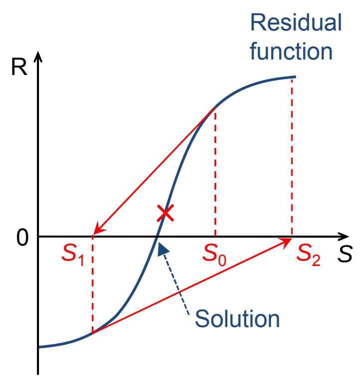
Stanford Center for Carbon Storage
Affiliate Meeting
May 21-22, 2013

Introduction

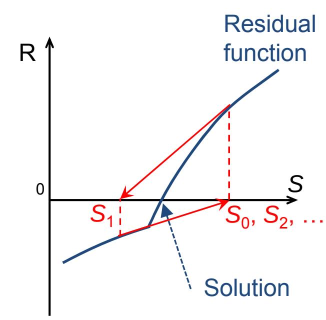
- Long-term objective:
  - Build an efficient nonlinear solver for coupled flow and transport in porous media, applicable in modeling CO_2 storage, oil recovery, etc.
- Previous works:
  - Potential ordering strategy
    (Kwok & Tchelepi, JCP 2007)
  - Localization technique
    (Lu and Beckner, SPE 2011)
  - Ordering with localization
    (Shahvali & Tchelepi, SPE 2013)
  - Trust-Region Chopping
    (Jenny et al., JCP 2009; Wang & Tchelepi, JCP 2013)
- Here, we offer a deep theoretical probe into the nonlinearity of the simulation problem

Why Non-convergence Occurs?

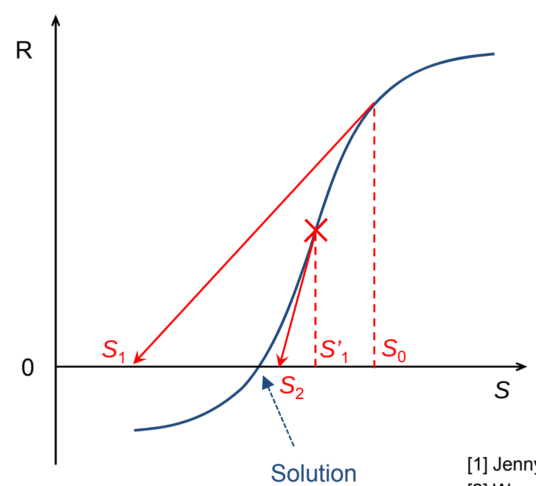
Due to the inflection point:



Due to the kink:



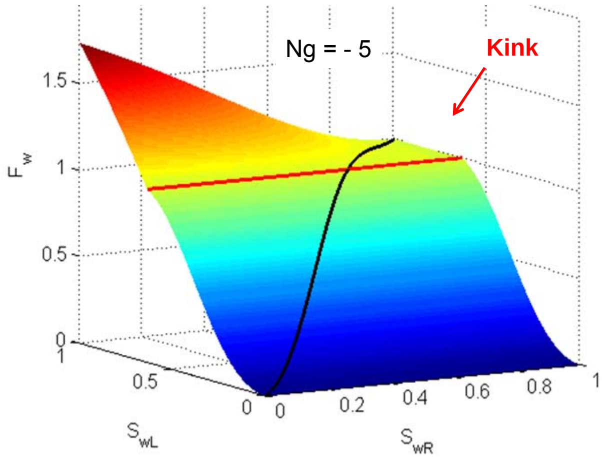
Trust-Region Chopping Method



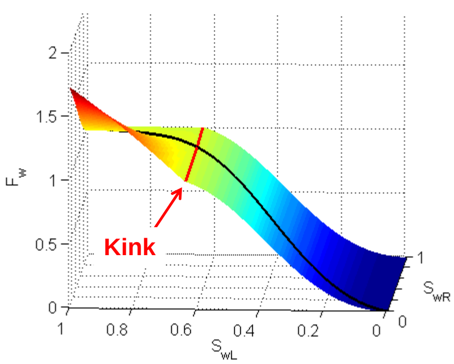
[1] Jenny et al., 2009
[2] Wang & Tchelepi, 2013
[3] Li & Tchelepi, 2014

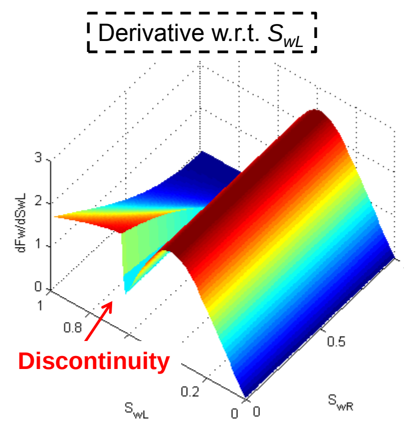
Numerical Flux (F)

Single-point upstream weighting is used.
We consider viscous and buoyancy forces here.



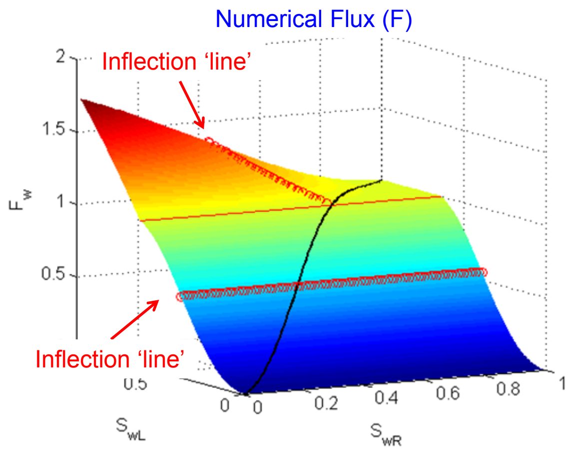
Numerical Flux (F)





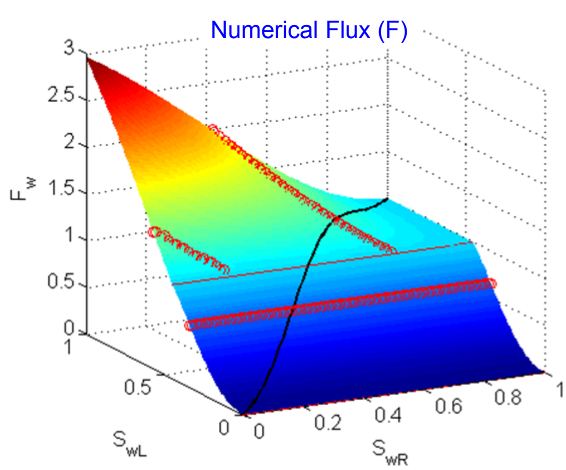
Theory of Inflection Lines

- For viscous and buoyancy forces (Ng = -5):



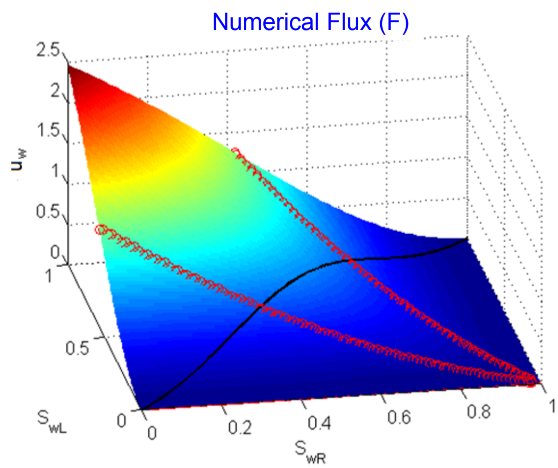
Inflection Lines on Numerical Flux

- Buoyancy is stronger (Ng = -10)



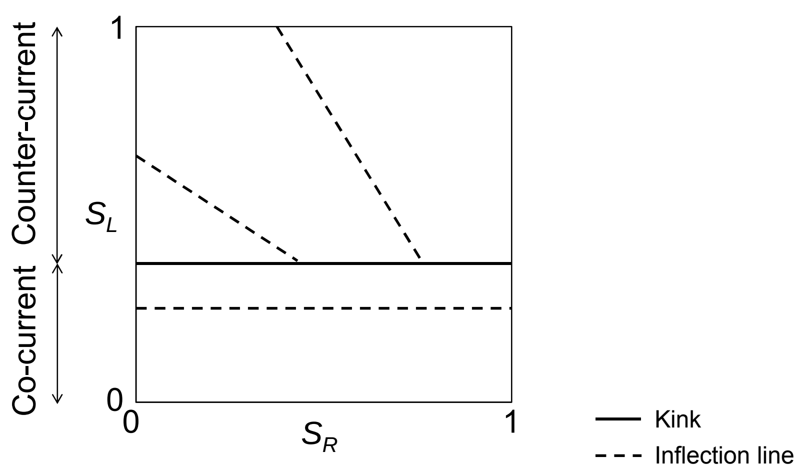
Inflection Lines on Numerical Flux

- Gravity segregation (Ng = $-\infty$):



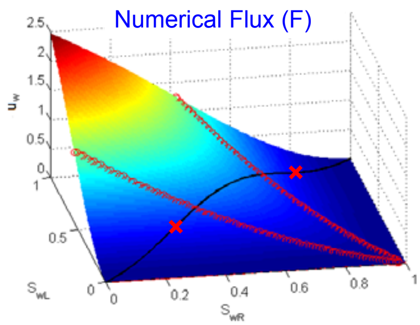
Chopping Strategies

- Locate the kink and inflection lines:

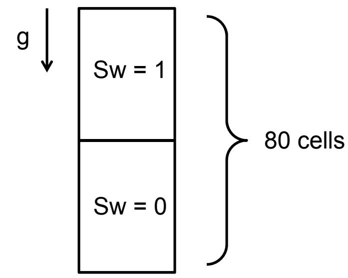


Numerical Example

Example: 1D gravity segregation

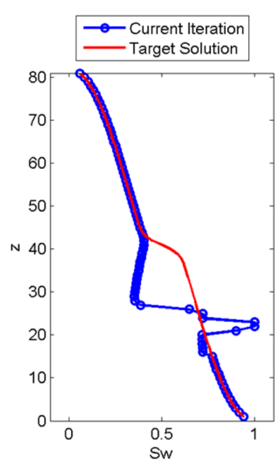


Initial condition:



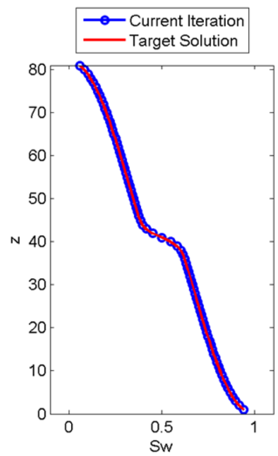
Convergence Analysis

- An example of convergence failure

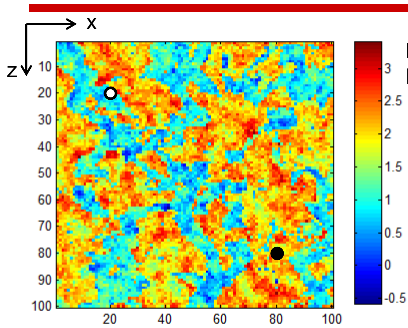


Convergence Analysis

- Our method:



2D Heterogeneous Model

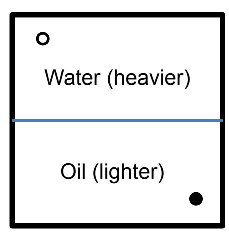


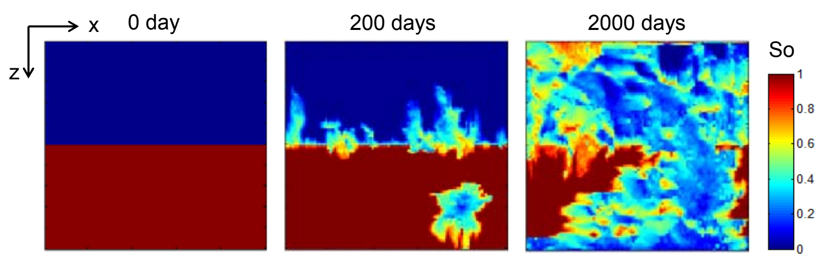
● Injector

○ Producer

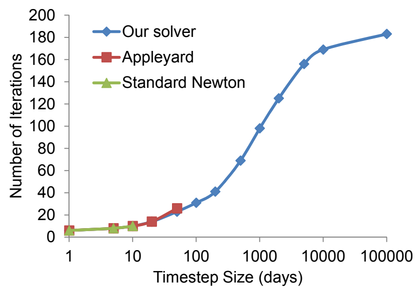
Injection rate:
1.4 CPVI / day

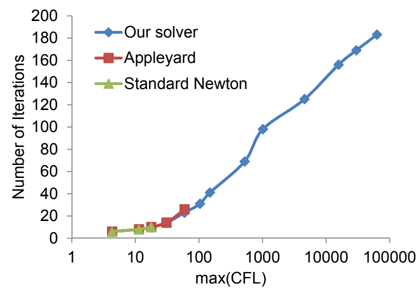
Initial condition:



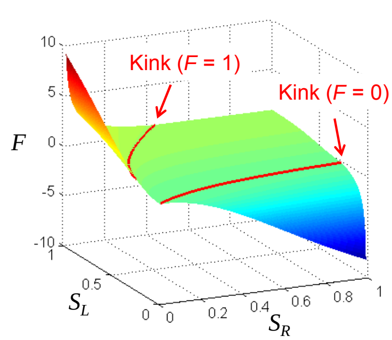


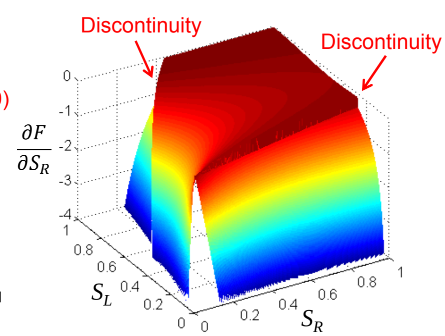
2D Heterogeneous Model





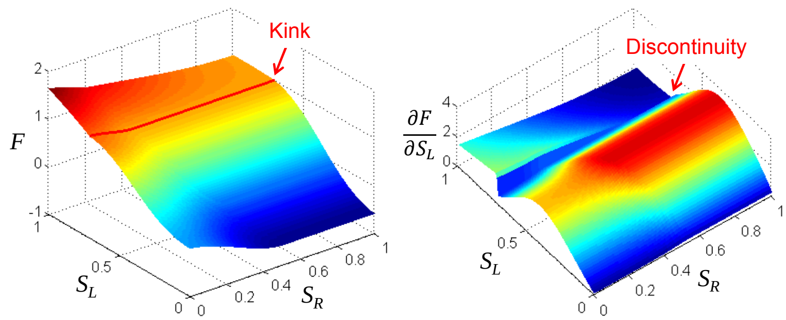
Nonlinearity of Capillarity



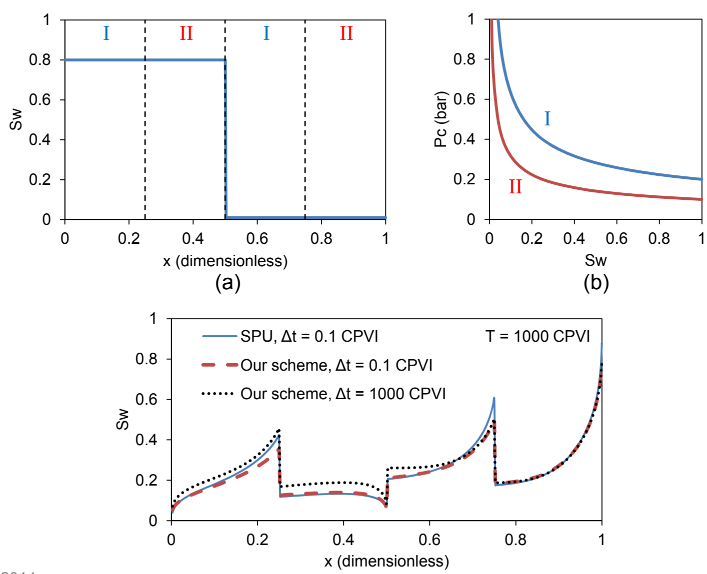


New Scheme

For viscous, buoyancy, and heterogeneous capillary forces:

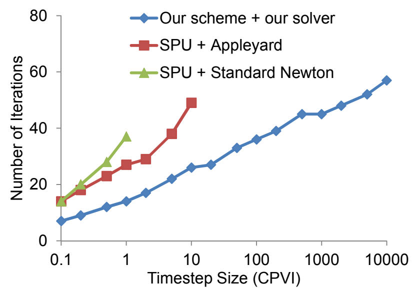


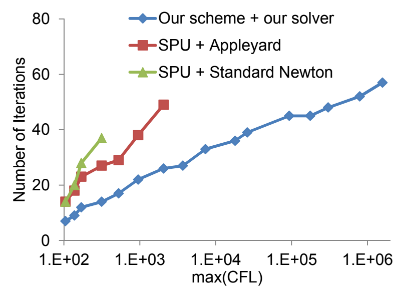
Numerical Example



(a) (b) (c)

Numerical Example





Conclusions

- The nonlinearity of the simulation problem can be understood by studying the nonlinear space of each cell interface
- Kinks and inflection lines cause convergence failure. They can be accurately located
- Our nonlinear solver guides Newton iterations to progress through kinks and inflection lines without causing oscillation and divergence
- Convergence is achieved for timesteps spanning several orders of magnitude
- **A clear understanding of the nonlinearity sparks innovations of nonlinear solvers and numerical schemes**

Acknowledgement

- GCEP (Global Climate & Energy Project)
- Xiaochen Wang
- Chevron ETC
- Brad Mallison