Data-Space Inversion for Reservoir Forecasting and Carbon Storage Monitoring

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Model Inversion

- Model inversion (history matching) is challenging
  - Complex geology (e.g., fractured reservoirs)
  - Nonlinear parameter-data relationship
  - Long turn-around time
- Data-space inversion approach is appealing when the history-matched model is less essential
  - More efficient
  - Can deal with complex models

Model Inversion versus Data-space Inversion

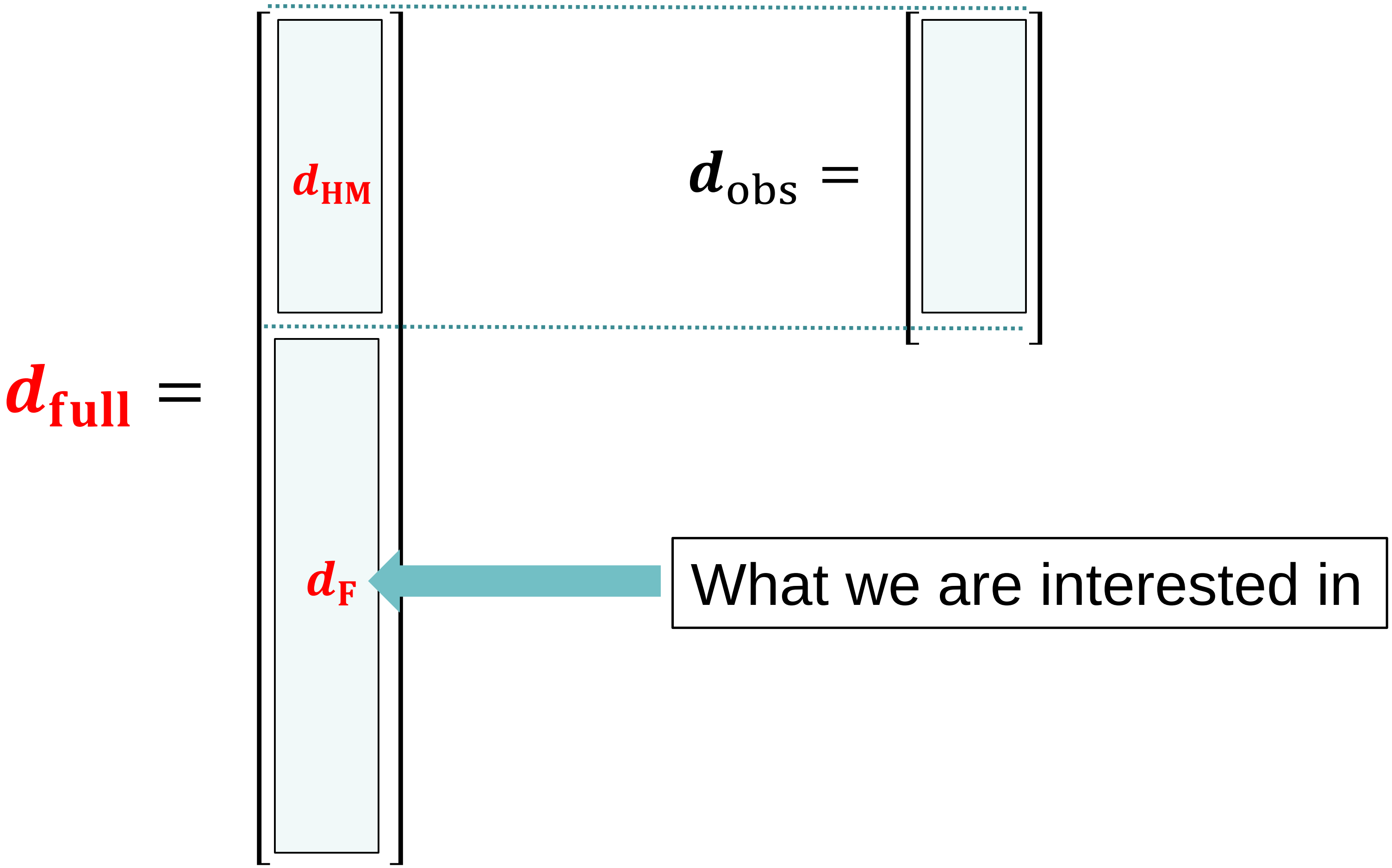
$$\boldsymbol{d}_{\textbf{full}} = \boldsymbol{g}(\boldsymbol{m})$$

$\boldsymbol{g}(\cdot)$: forward model

$\boldsymbol{d}_{\textbf{full}}$: data variables, such as oil production rate

$\boldsymbol{m}$: model parameters

Vector of Data Variables



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Under Bayesian Framework

Model inversion: **first** sample posterior model parameters

$$\boldsymbol{P}(\boldsymbol{m}|\boldsymbol{d}_{\text{obs}}) = \text{const} \times \boldsymbol{P}(\boldsymbol{d}_{\text{obs}}|\boldsymbol{m})\boldsymbol{P}(\boldsymbol{m}) \rightarrow \boldsymbol{d}_{\text{full}} = \boldsymbol{g}(\boldsymbol{m})$$

Data-space inversion: **directly** sample posterior forecasts

$$\boldsymbol{P}(\boldsymbol{d}_{\text{full}}|\boldsymbol{d}_{\text{obs}}) = \text{const} \times \boldsymbol{P}(\boldsymbol{d}_{\text{obs}}|\boldsymbol{d}_{\text{full}})\boldsymbol{P}(\boldsymbol{d}_{\text{full}})$$

Data-space Inversion (DSI)

$$P(\boldsymbol{d}_{\text{full}}|\boldsymbol{d}_{\text{obs}}) = \text{const} \times P(\boldsymbol{d}_{\text{obs}}|\boldsymbol{d}_{\text{full}})P(\boldsymbol{d}_{\text{full}})$$

- Likelihood function

$$P(\boldsymbol{d}_{\text{obs}}|\boldsymbol{d}_{\text{full}}) \propto \mathbf{exp}\left(-\frac{1}{2}(\boldsymbol{H}\boldsymbol{d}_{\text{full}} - \boldsymbol{d}_{\text{obs}})^T \boldsymbol{C}_D^{-1}(\boldsymbol{H}\boldsymbol{d}_{\text{full}} - \boldsymbol{d}_{\text{obs}})\right)$$

$\boldsymbol{H}$: selection matrix

$\boldsymbol{C}_D$: covariance matrix for measurement error

- Prior probability density function (PDF)

$$P(\boldsymbol{d}_{\text{full}}) \propto \mathbf{exp}\left(-\frac{1}{2}(\boldsymbol{d}_{\text{full}} - \boldsymbol{d}_{\text{mean}})^T \boldsymbol{C}_{\text{pd}}^{-1}(\boldsymbol{d}_{\text{full}} - \boldsymbol{d}_{\text{mean}})\right)$$

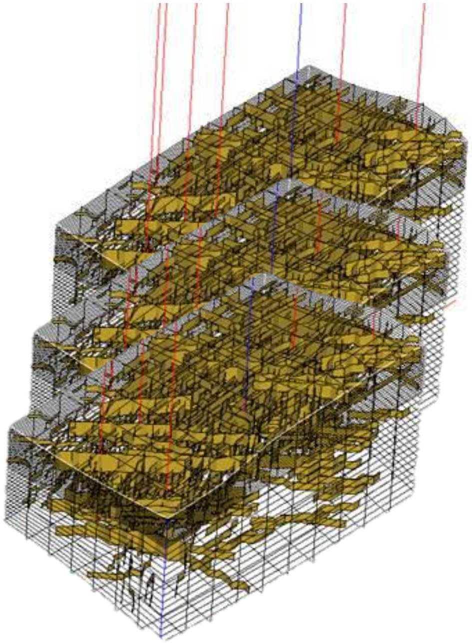
$\boldsymbol{d}_{\text{mean}}$: prior mean of data variables ($\boldsymbol{d}_{\text{full}}$)

$\boldsymbol{C}_{\text{pd}}$: prior covariance matrix for data variables

What if $\boldsymbol{d}_{\text{full}}$ are not Gaussian a priori? **Reparameterization!**

Data-space Inversion Procedure: Step 1

- Generate multiple prior realizations $\boldsymbol{m}_i$



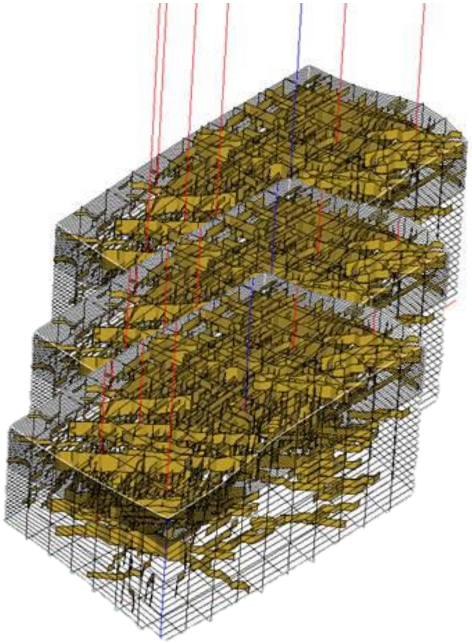
Step 2: Flow Simulation

- Generate multiple prior realizations $\boldsymbol{m}_i$
- Perform flow simulation: $(\boldsymbol{d}_{\mathbf{full}})_{\boldsymbol{i}} = \boldsymbol{g}(\boldsymbol{m}_i)$

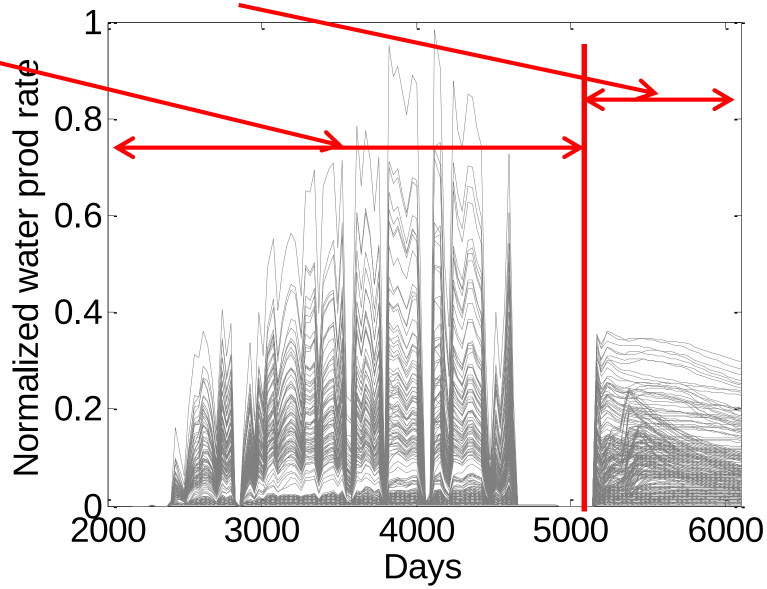
$$(\boldsymbol{d}_{\mathbf{full}})_{\boldsymbol{i}} = [\underbrace{d_1, \cdots, d_{\mathrm{N_h}}}_{\text{History-matching Period}}, \underbrace{d_{\mathrm{N_h}+1}, \cdots, d_{\mathrm{N_F}}}_{\text{Prediction Period}}]_i^T$$

History-matching Period

Prediction Period

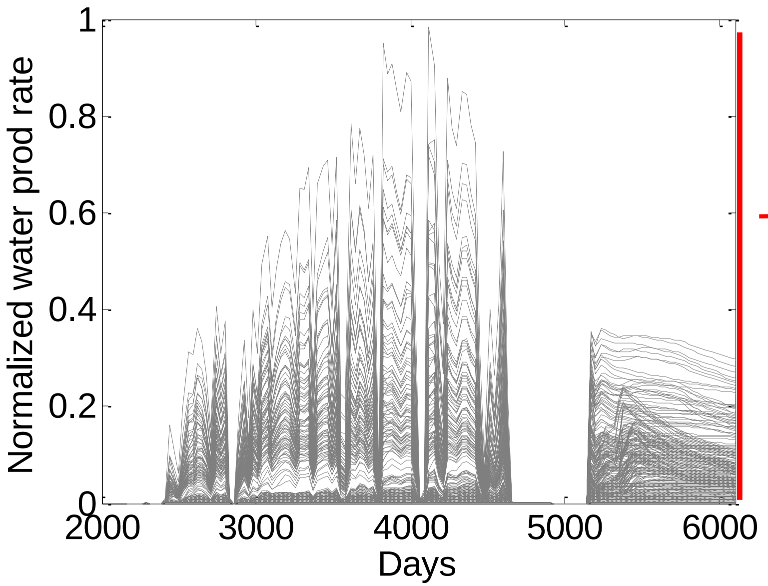


Expensive, but parallelizable

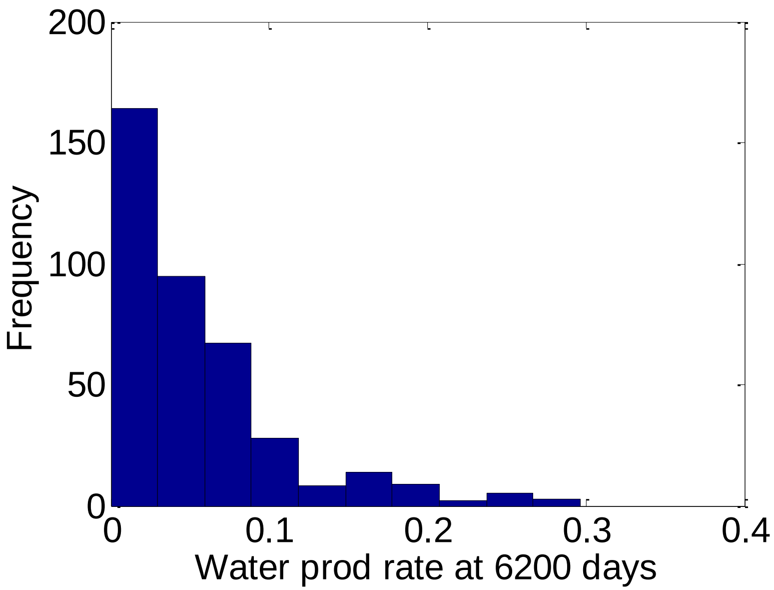


Step 3: Reparameterize Data Variables

- Non-Gaussian data variables $\boldsymbol{d}_{\mathbf{full}}$



Non-Gaussian histogram



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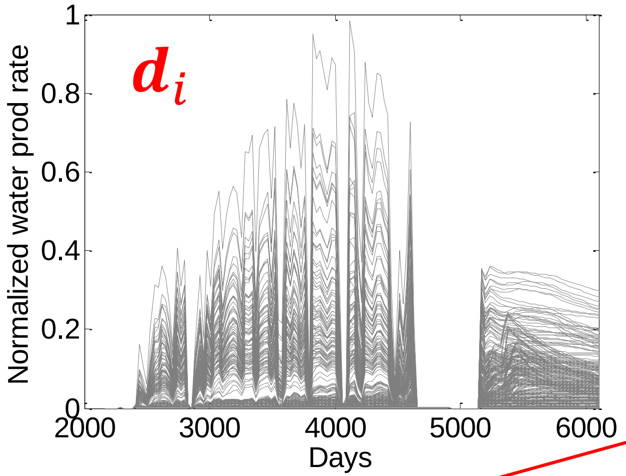
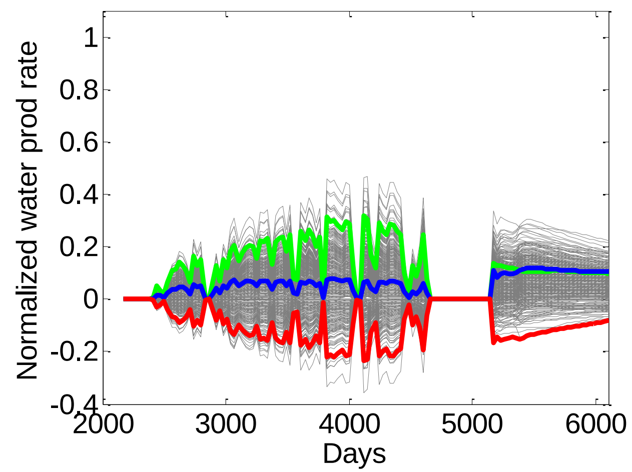


Principal Component Analysis

Basis matrix $\boldsymbol{\Phi}$
Mean data vector $\boldsymbol{d}_{\text{mean}}$

Parameters to be estimated

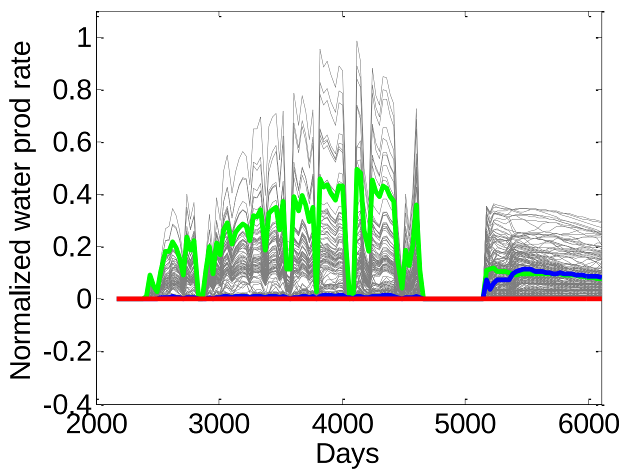
$$\boldsymbol{d}_{\text{PCA}} = \boldsymbol{\Phi}\boldsymbol{\xi} + \boldsymbol{d}_{\text{mean}}$$



Histogram Transformation

$\boldsymbol{h}_{\text{T}}(\cdot)$

$$\boldsymbol{d}_{\text{full}} = \boldsymbol{f}(\boldsymbol{\xi}) = \boldsymbol{h}_{\text{T}}(\boldsymbol{d}_{\text{PCA}})$$



Data-space Inversion with New Parameters

- Posterior PDF of new parameters

$$\boldsymbol{P}(\boldsymbol{\xi}|\boldsymbol{d}_{\text{obs}}) = \text{const} \times \boldsymbol{P}(\boldsymbol{d}_{\text{obs}}|\boldsymbol{\xi})\boldsymbol{P}(\boldsymbol{\xi})$$

- Likelihood function

$$\boldsymbol{P}(\boldsymbol{d}_{\text{obs}}|\boldsymbol{\xi}) \propto \mathbf{exp}\left(-\frac{1}{2}(\boldsymbol{H}\boldsymbol{f}(\boldsymbol{\xi}) - \boldsymbol{d}_{\text{obs}})^T \boldsymbol{C}_D^{-1}(\boldsymbol{H}\boldsymbol{f}(\boldsymbol{\xi}) - \boldsymbol{d}_{\text{obs}})\right)$$

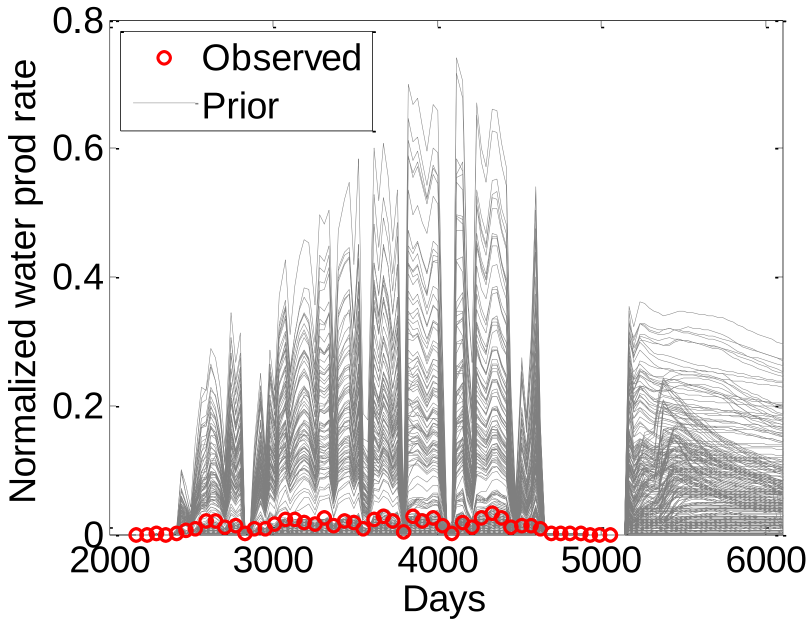
- Prior PDF ($\boldsymbol{\xi}$ is standard normal from PCA)

$$\boldsymbol{P}(\boldsymbol{\xi}) \propto \mathbf{exp}\left(-\frac{1}{2}\boldsymbol{\xi}^T\boldsymbol{\xi}\right)$$

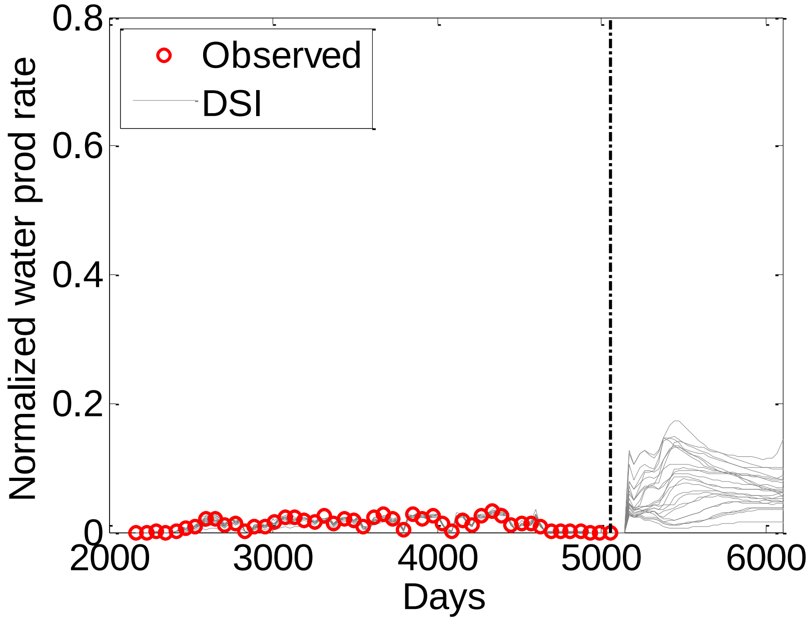
Step 4: Generate Posterior Forecasts

- Apply randomized maximum likelihood (RML) method in the data space

Objective function: $S(\boldsymbol{\xi}) = (\boldsymbol{Hf}(\boldsymbol{\xi}) - \boldsymbol{d}_{\text{obs}}^{*})^{T} \boldsymbol{C}_D^{-1} (\boldsymbol{Hf}(\boldsymbol{\xi}) - \boldsymbol{d}_{\text{obs}}^{*}) + (\boldsymbol{\xi} - \boldsymbol{\xi}^{*})^{T} (\boldsymbol{\xi} - \boldsymbol{\xi}^{*})$



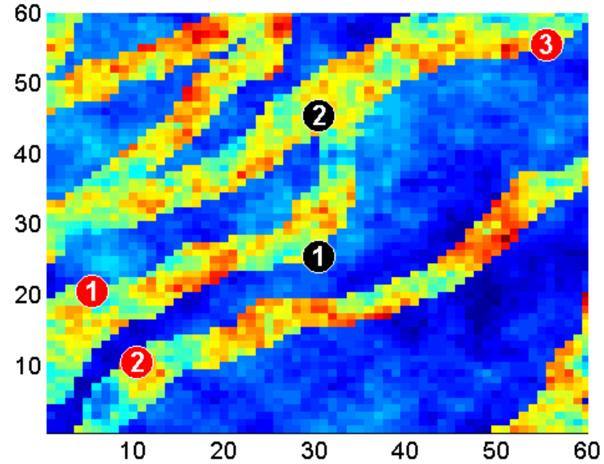
Before matching data (prior)

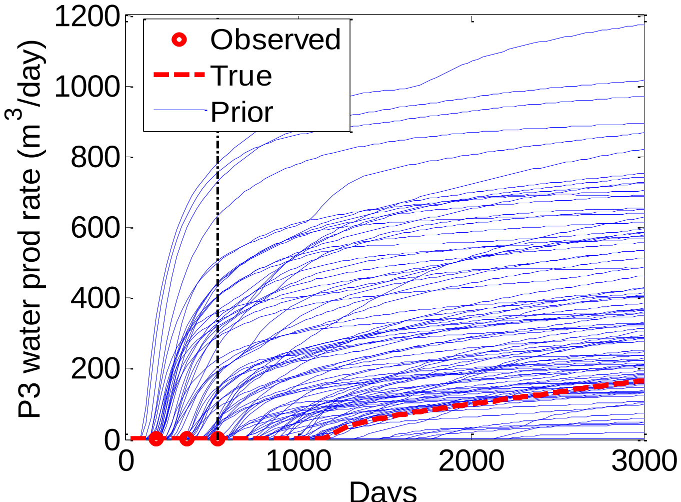


After matching data (posterior)

Two-dimensional Channelized System

- Oil-water with AD-GPRS
- 2D model, 60 x 60
- 2 injectors, BHP controlled
- 3 producers, BHP controlled
- History: first 540 days
  - Observed data available at I2, P3
- DSI performs 500 simulations

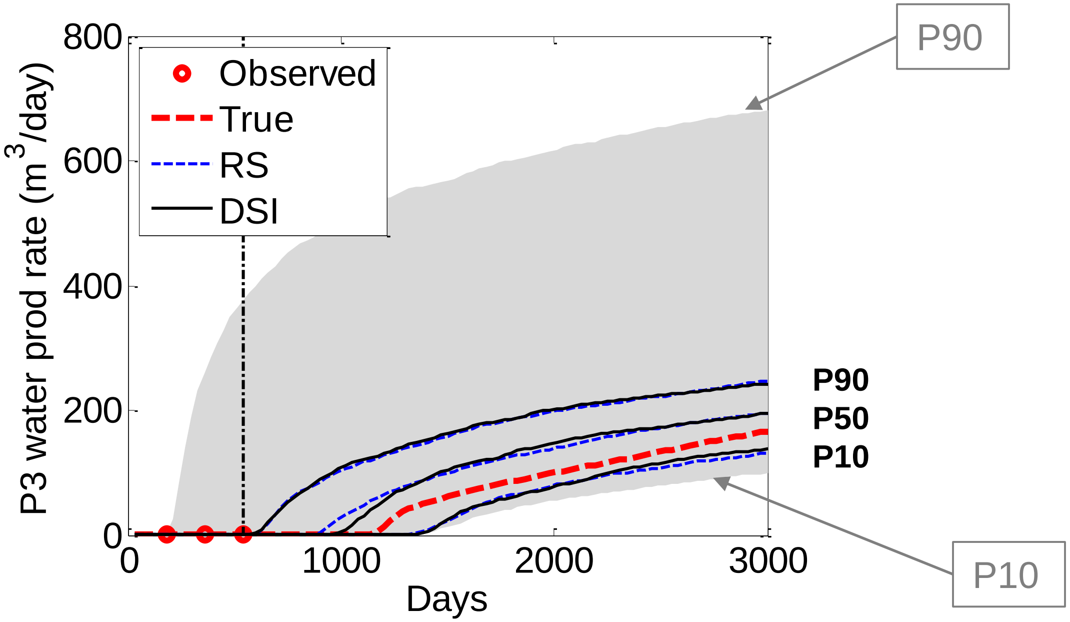




(Forecasts from prior models)

Comparison with Rejection Sampling

DSI shown to provide accurate uncertainty quantification for a bi-modal channelized system



RS: 1,000,000 prior simulations; 242 accepted

DSI: 500 prior simulations; 100 posterior predictions

(results from Sun and Durlofsky, 2017)

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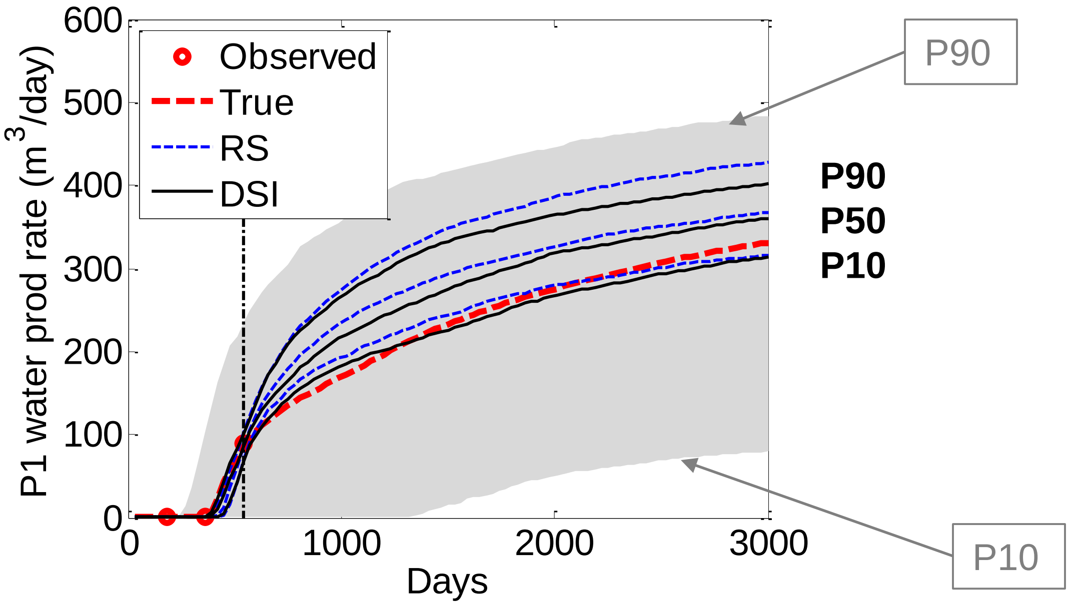

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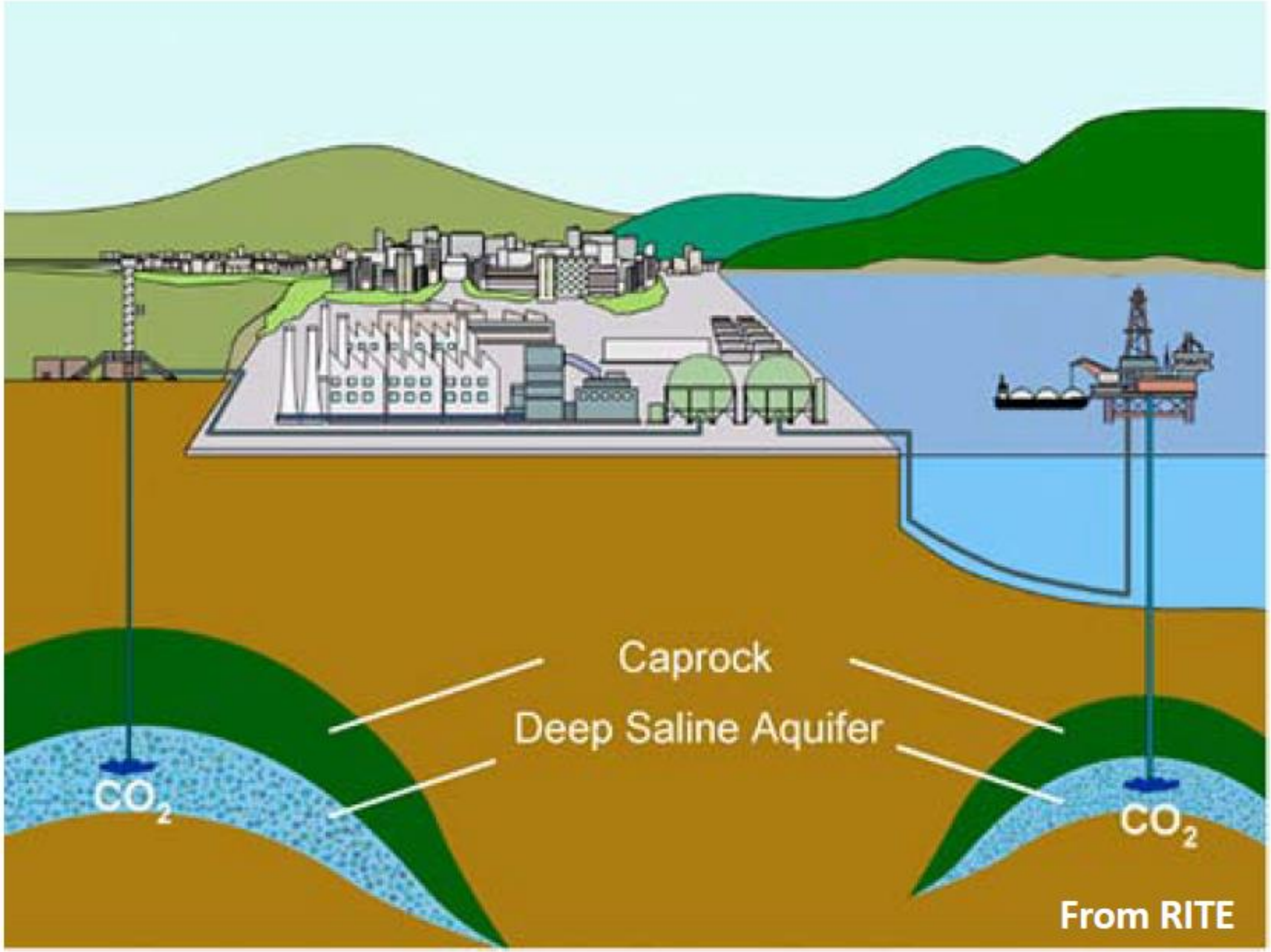
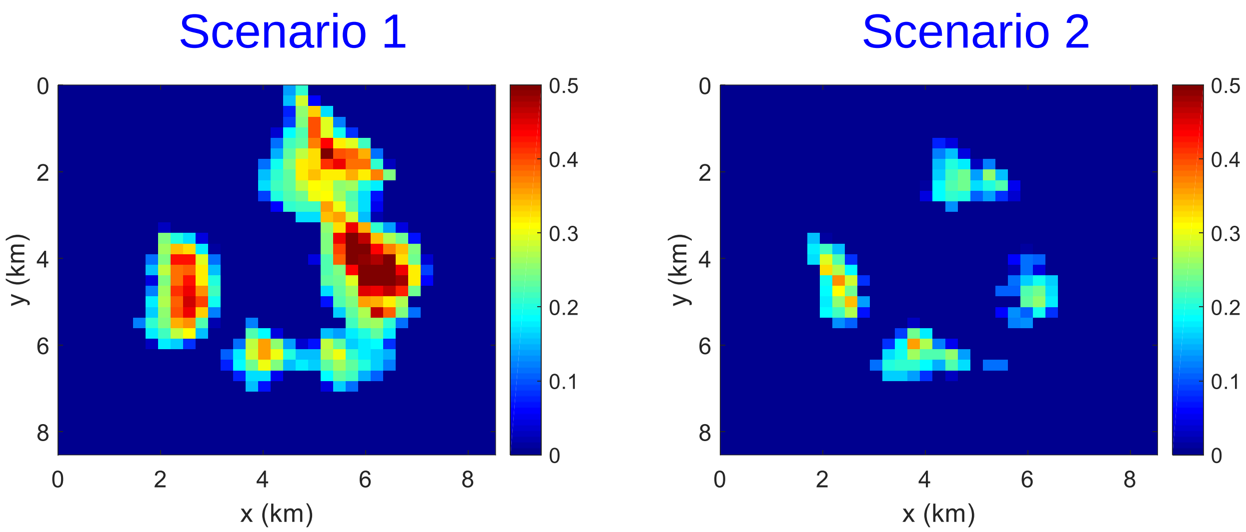
Carbon Capture and Storage (Schematic)



Research Objectives

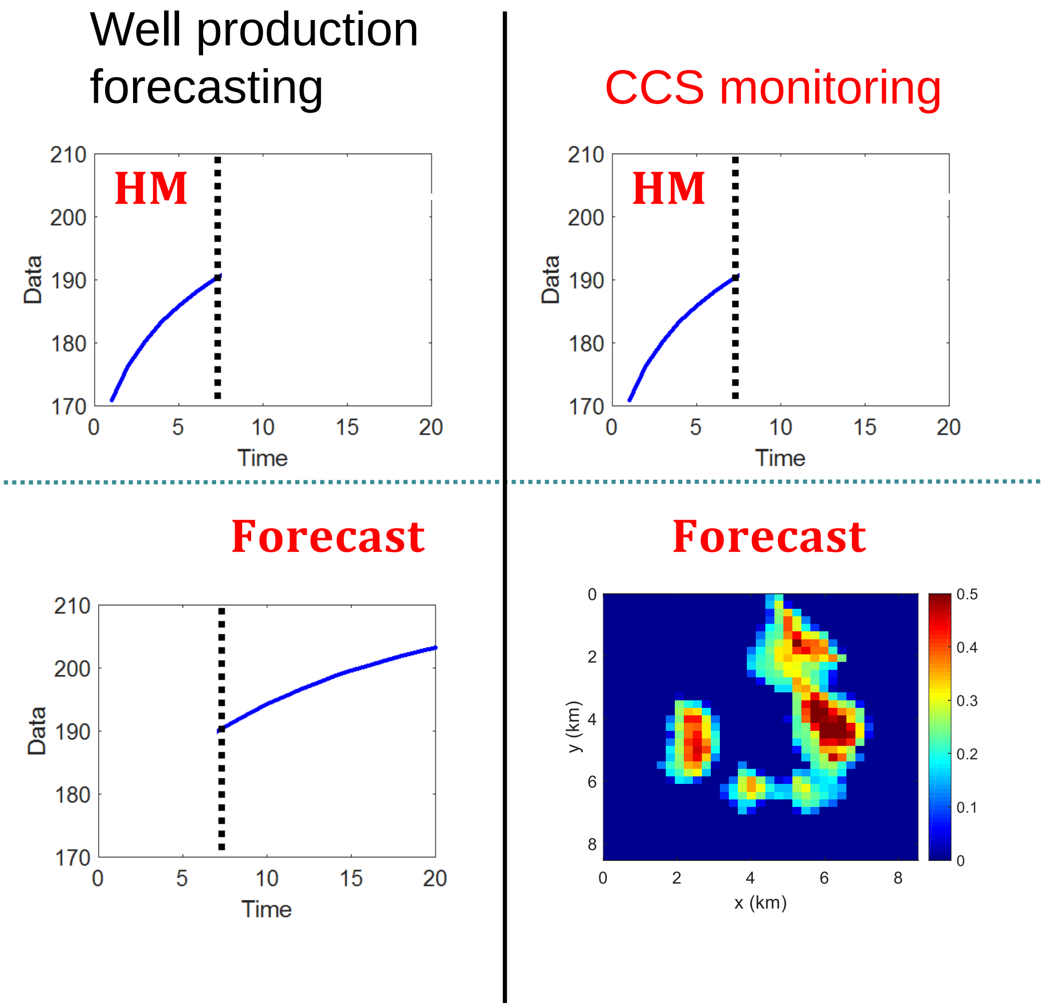
- Predict CO_2 plume under caprock (e.g., 200 years)



- Optimize placement of monitoring wells (MWs)

CO_2 Plume Prediction with DSI

$$d_{full} = \begin{bmatrix} d_{HM} \\ d_F \end{bmatrix}$$

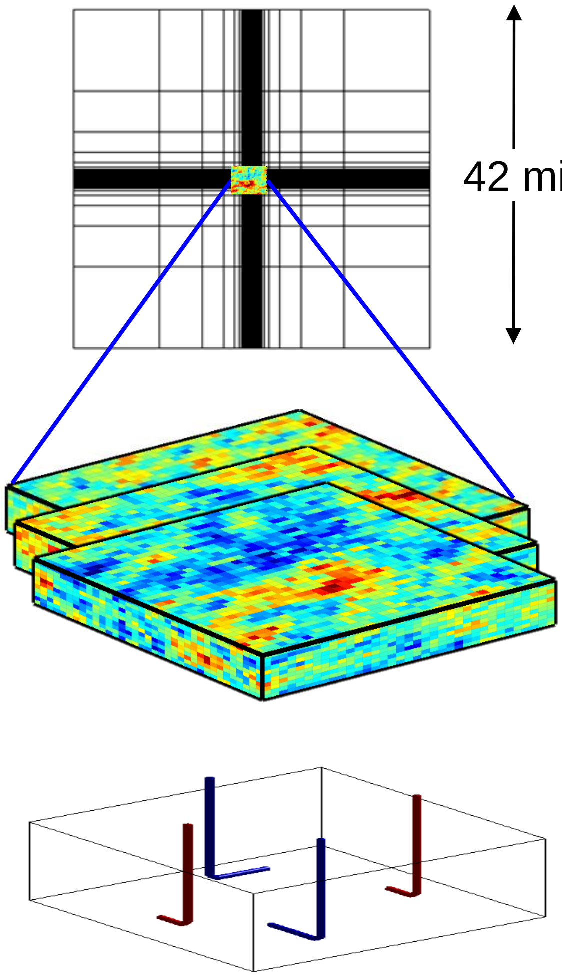


Aquifer Model

- Inject 2 Mt/year for 20 years
- Equilibrate for 180 years
- Storage aquifer: 5 mi x 5 mi x 500 ft
- 4 horizontal injectors
- Heterogeneous field

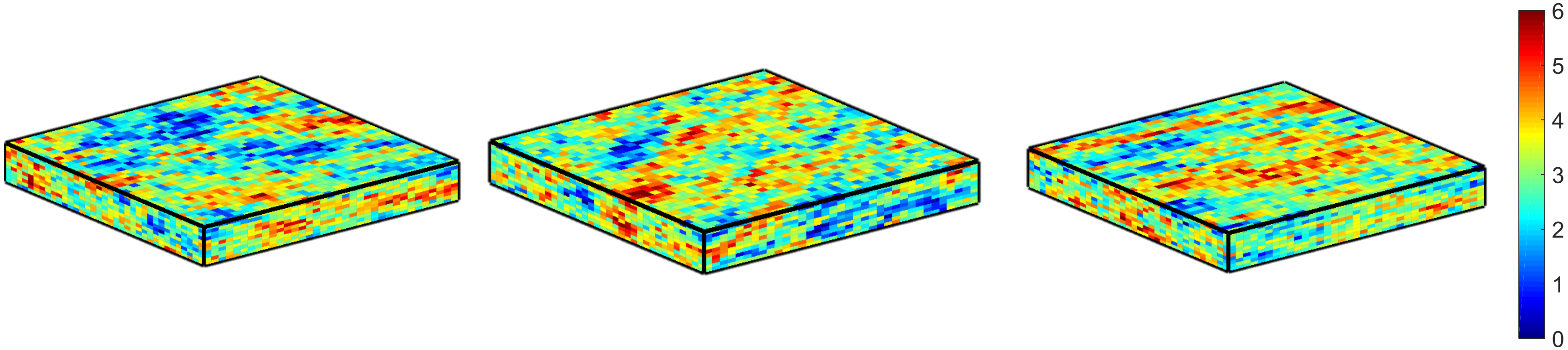
$$\ln(k) = a\phi + b + \text{noise}$$

- Physics included
  - Relative permeability hysteresis
  - Heterogeneous P_c



Model Uncertainty

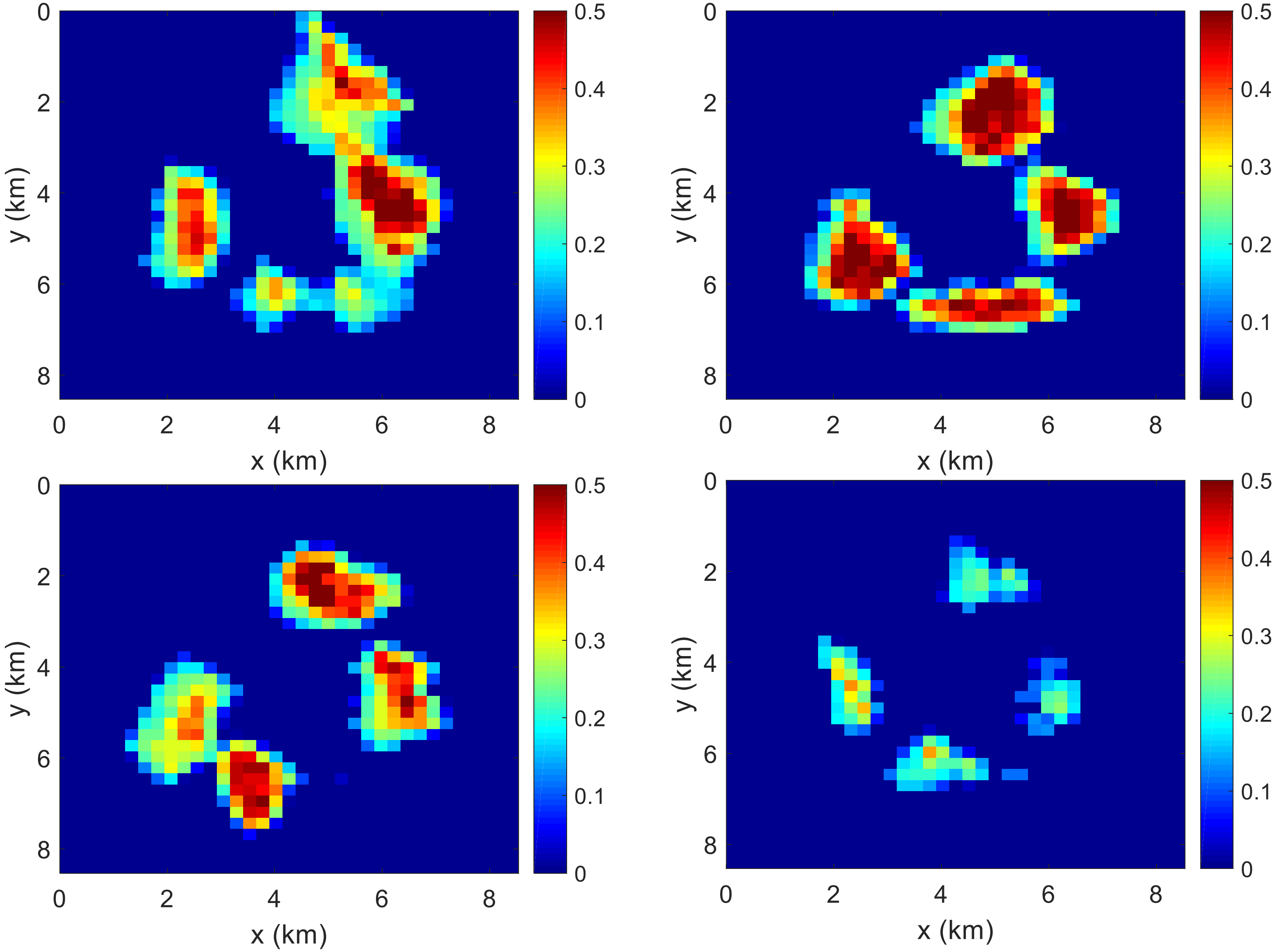
- ϕ, k realizations with different variograms



- Uncertain large-scale pressure gradient

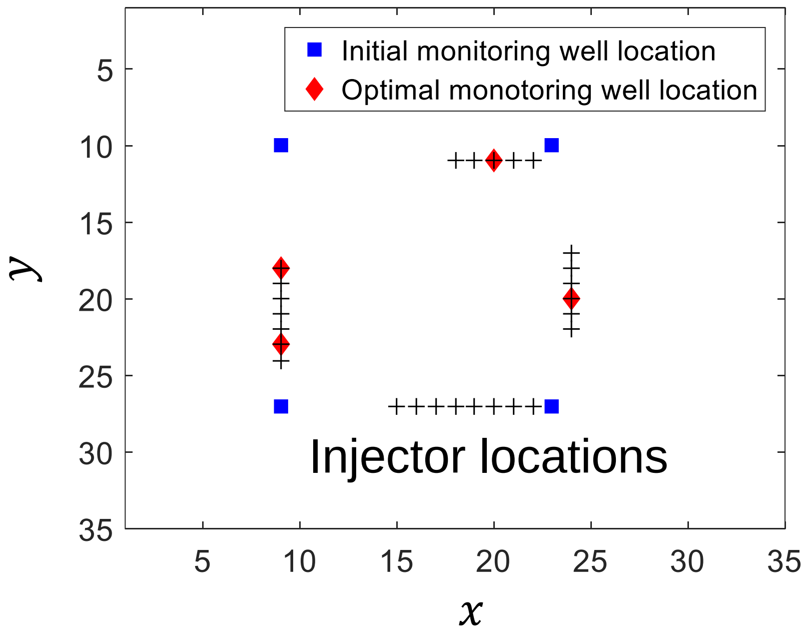
- Uncertain k_z/k_x

Possible CO_2 Plume from Prior Models



Optimal Monitoring Well Locations

- Observations: yearly CO_2 saturation and pressure (4 MWs, 480 data points)
- Adapt ensemble variance analysis (Jincong He et al. 2017) for MW locations optimization



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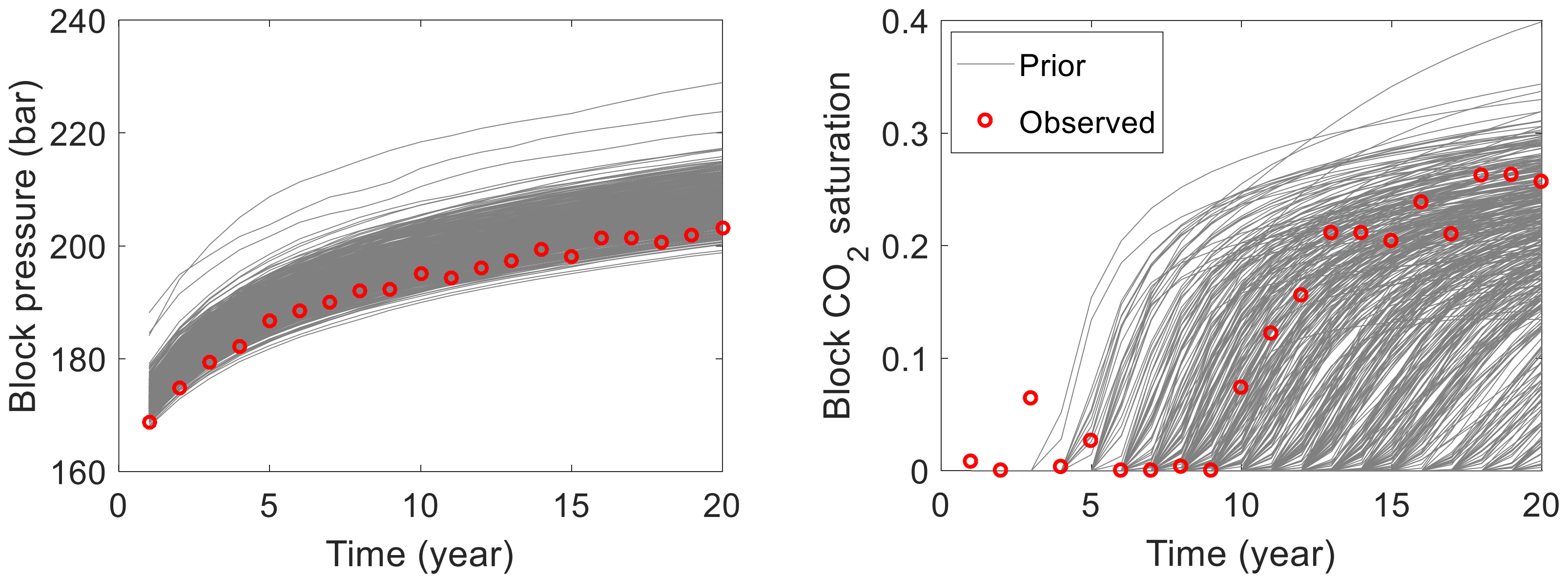
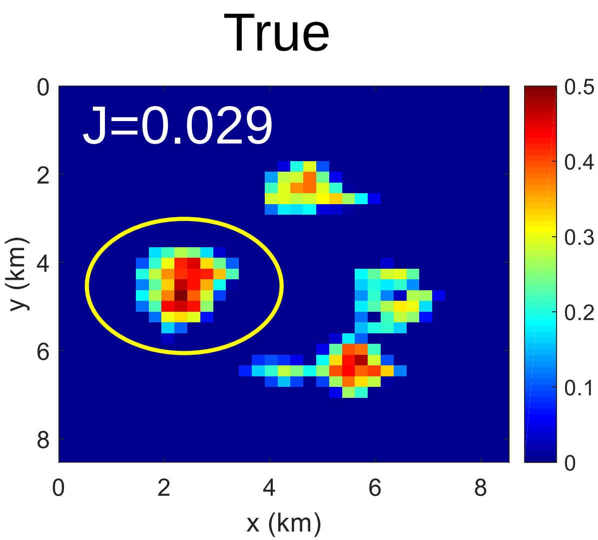
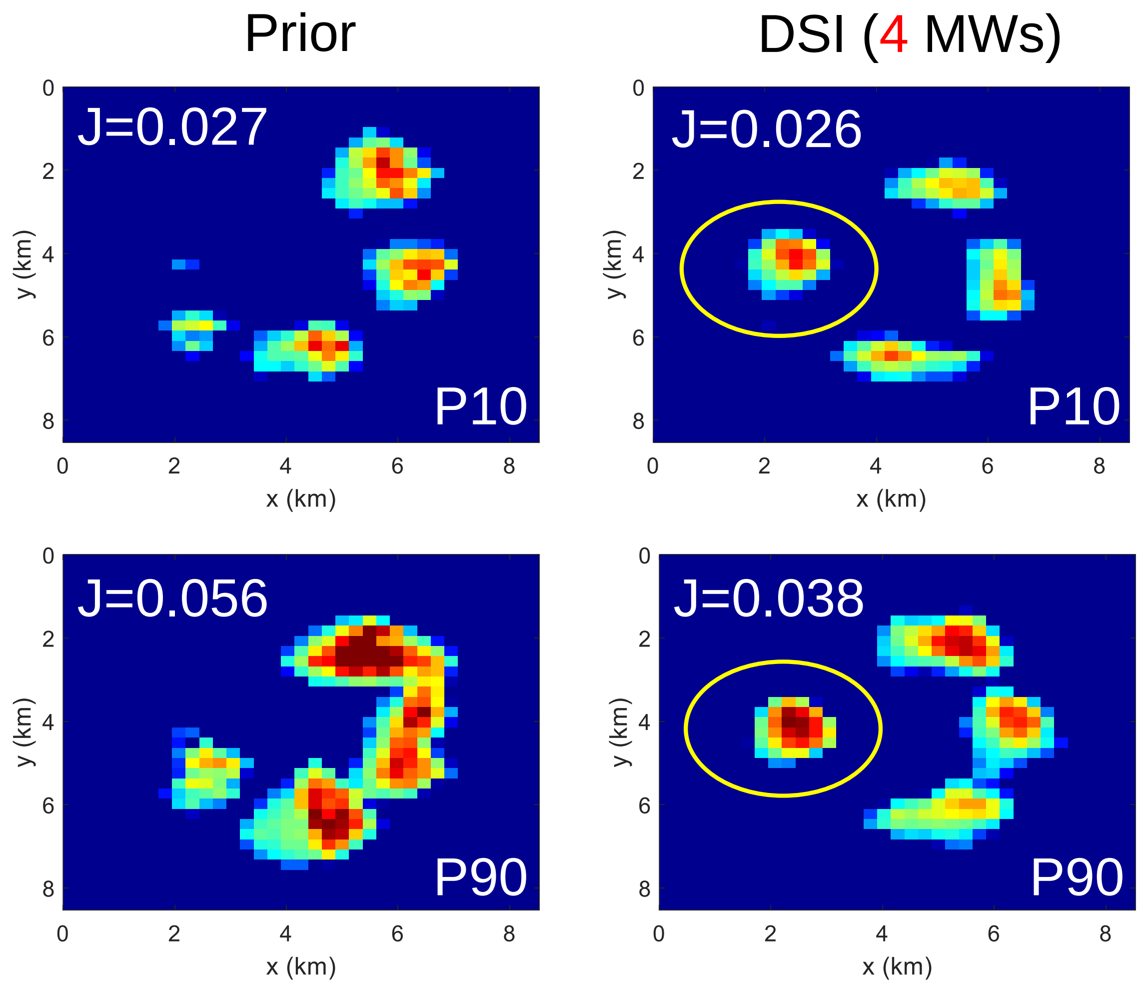


Results with 4 MWs (Optimal Placement)

Top layer plume at 200 years

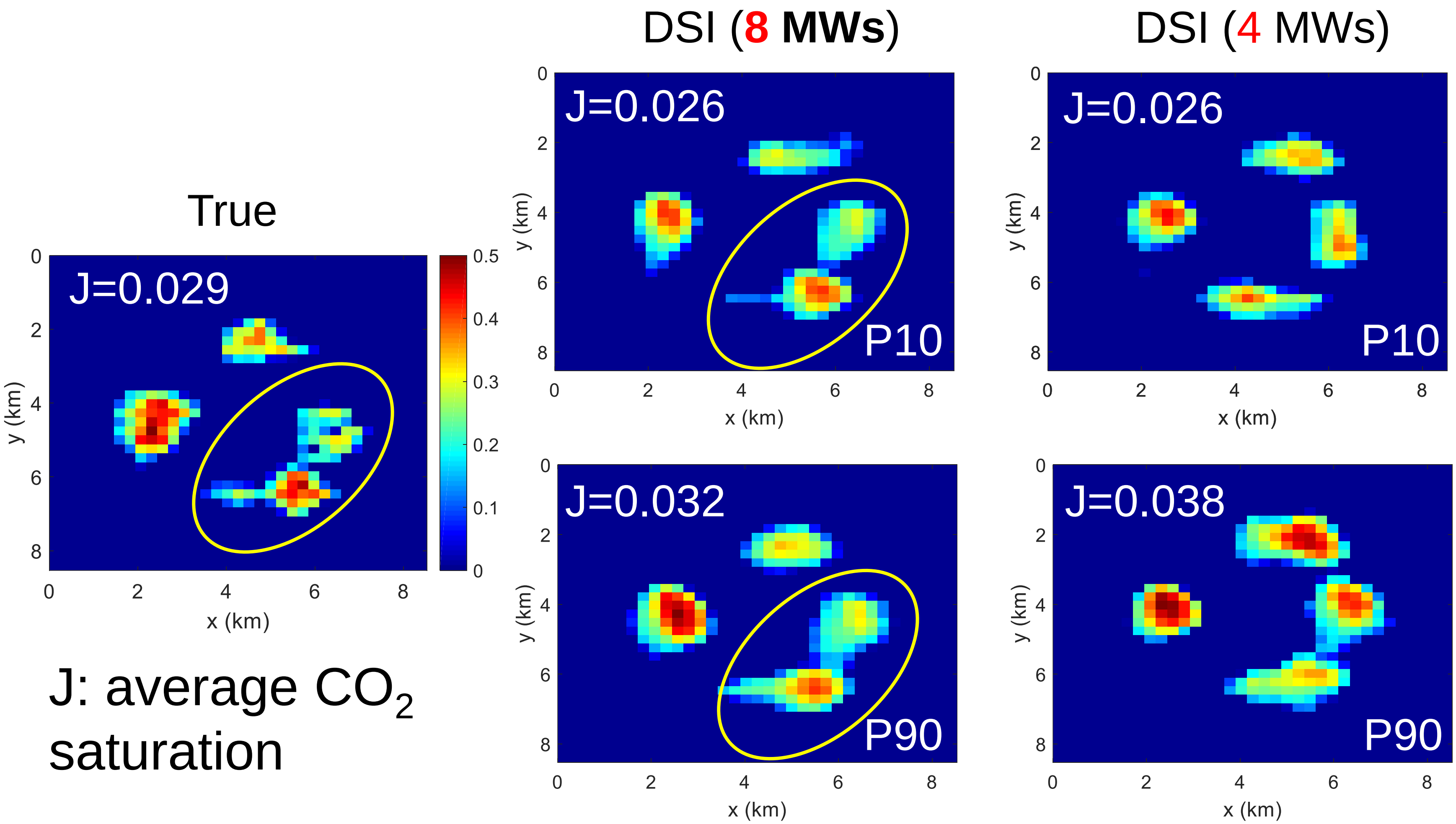




J: average CO_2 saturation

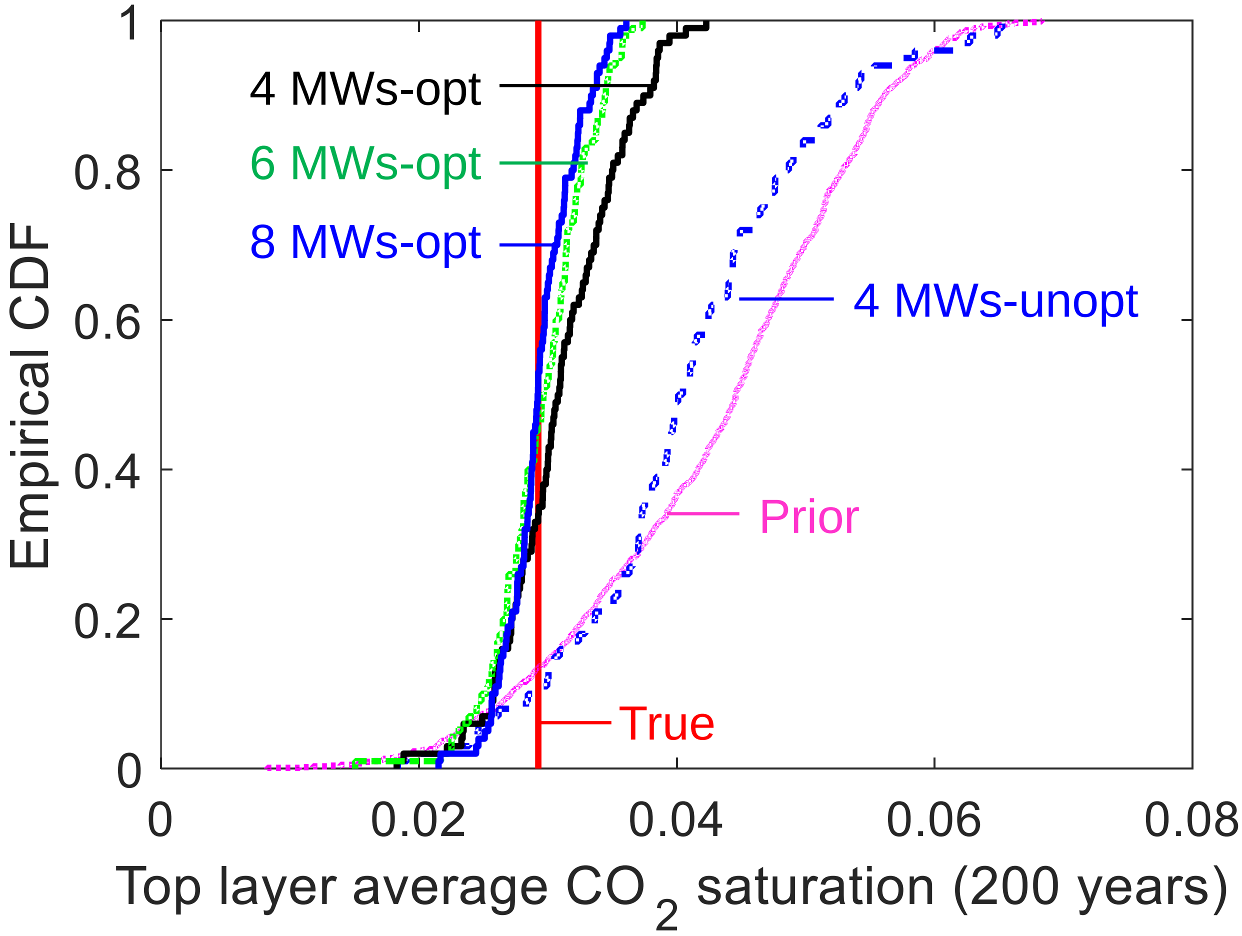
Results with More Monitoring Wells

Top layer plume at 200 years



J: average CO_2 saturation

CDF for Average CO_2 in Top Layer



Conclusions

- Apply DSI procedures for uncertainty quantification in subsurface flow problems
  - Conventional reservoirs
  - Naturally fractured reservoirs
  - CO_2 storage
- DSI posterior uncertainty (P10, P50, P90) agrees well with rejection sampling
- Demonstrated clear uncertainty reduction in CO_2 plume location

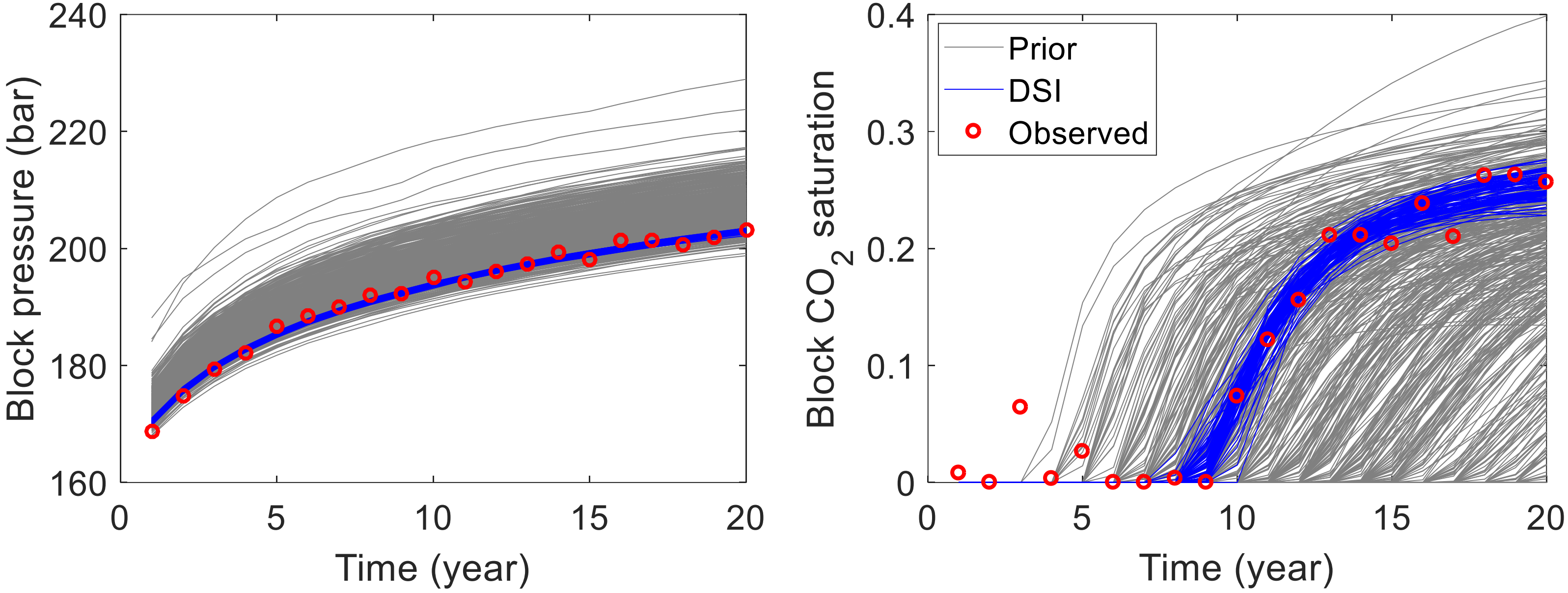
Acknowledgments

- Chevron ETC (CoRE) for funding
- Robin Hui and Jairam Kamath (Chevron)
- Larry Zhaoyang Jin, Dave Cameron
- Oleg Volkov
- Stanford Center for Computational Earth & Environmental Science (CEES)

Thanks

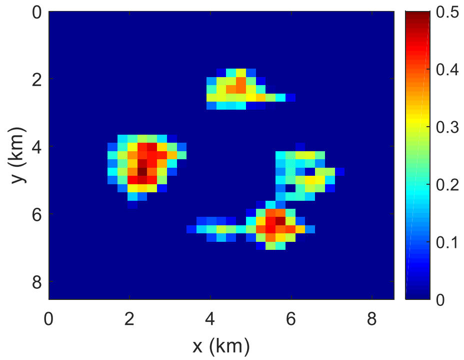
Questions?

DSI Prediction

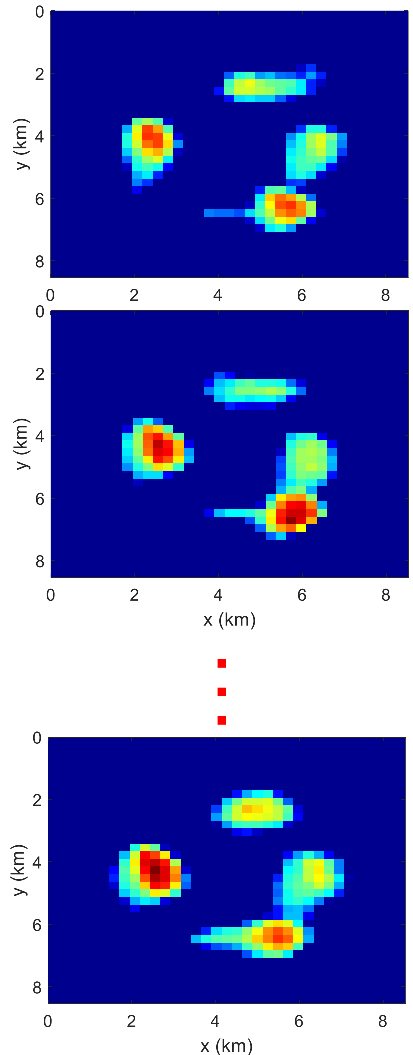


Compare Plume Prediction

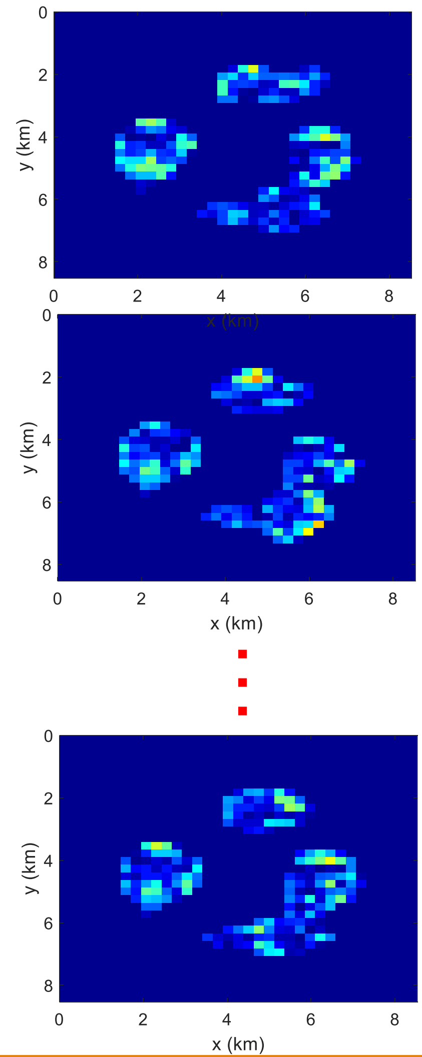
True Model



DSI results

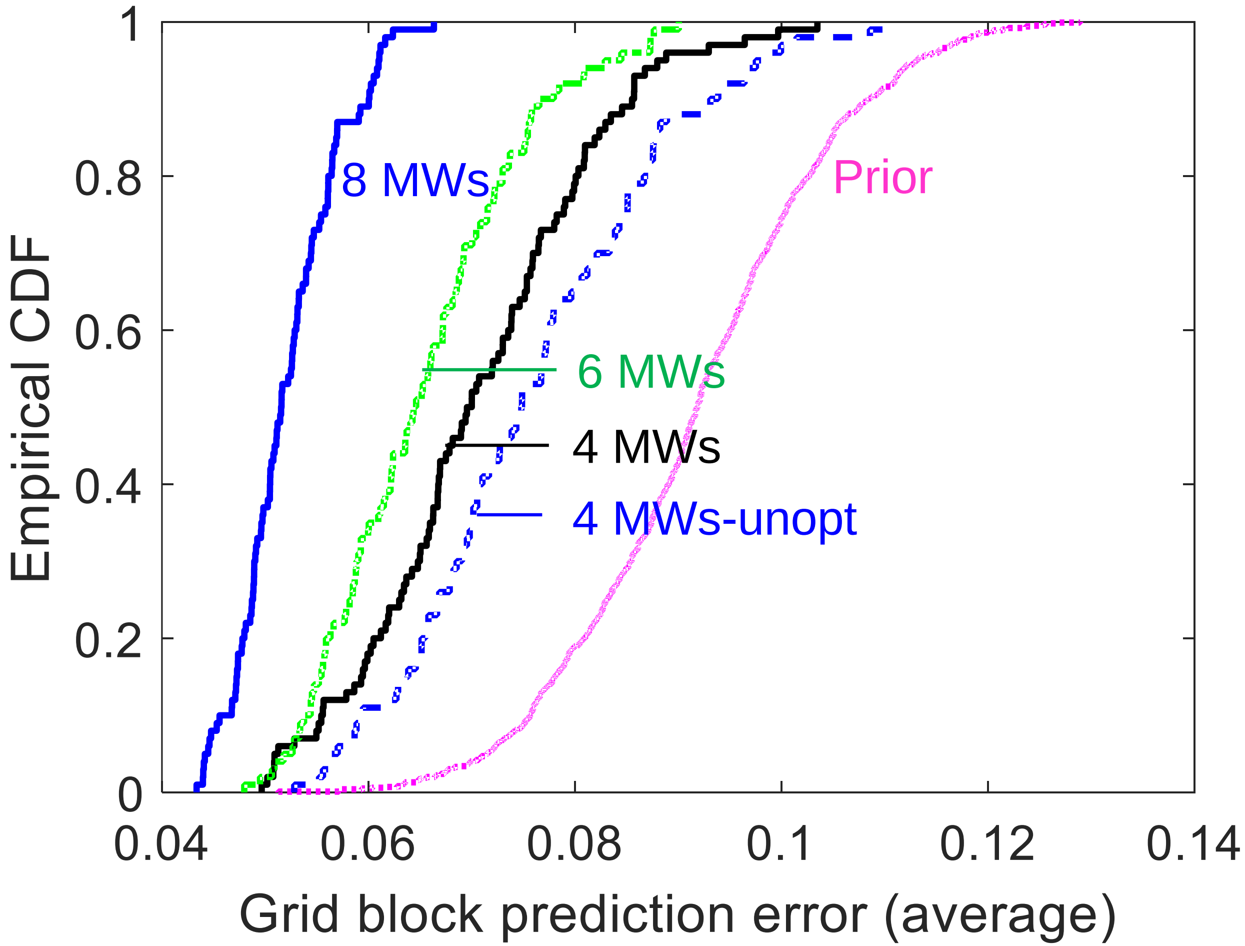


|DSI – True|$_1$



Compute mean error over all realizations

Pixel-wise Prediction Error (Average)



Another Test Case (8 MWs)

True Model